Test 2 sessi 0708

FIZIK IV (MODERN PHYSICS)

ZCT 104

TEST 2

15 March 2008

Instruction: Answer both questions on blank paper. Explain your steps as clearly as possible.

1. In Compton scattering experiment, X-ray photons with frequency f are incident on a target. After collision with electrons which initially at rest, the scattered photons with frequency of f' are observed at angle θ while the recoiled electrons are observed at angle ϕ.
   (a) What is the kinetic energy of the recoiling electron in terms of the f and f'?
   (b) Show that the kinetic energy of the recoiling electron in the Compton scattering is give by the general formula:

$$KE_{recoil\ electron} = \frac{2\left(\frac{h}{m_e c}\right)\sin^2\left(\frac{\theta}{2}\right)}{\frac{c}{f} + 2\left(\frac{h}{m_e c}\right)\sin^2\left(\frac{\theta}{2}\right)} hf$$

   Where m_e = rest mass of electron
   h = Planck's constant
   c = speed of light
   θ = angle of the scattered-photon
   f = frequency of the incident photon

   [*Hint:* $\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$.]

   (c) Show that the maximum kinetic energy of the recoiling electron in this Compton scattering experiment is given by

$$KE_{max}(recoiling\ electron) = hf \frac{\frac{2hf}{m_e c^2}}{1 + \frac{2hf}{m_e c^2}}$$

   (i) At what angles θ and ϕ does this occur?
   (ii) If we detect a scattered electron at ϕ = 0° of 100 keV, what energy photon was scattered?

[25 marks]

2. Being not prohibited by the uncertainty principle, a particle of mass m can be created spontaneously in vacuum, exists for a short period of lifetime, Δt, and then annihilate into vacuum again. **[25 marks]**

   (i) How long will this particle exist (i.e, what is Δt in terms of m)?

   (ii) If the particle travels at approximately the speed of light, approximately how far will it travel during its short existence?
   (iii) If such a spontaneous creation-annihilation happens within the nucleus, of a typical size of r ~ fm, estimate the mass of m in unit of MeV.

(iv) Estimate the lifetime of *m* of the mass as calculated in (iii), in unit of seconds.

Solution:

1.

(a) $$KE_{recoil\ electron} = hf - hf^{'} \qquad (1a)$$

(b) From the Compton scattering formula, we have

$$\lambda^{'} - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \qquad (1b)$$

By substituting

$$\lambda^{'} = \frac{c}{f^{'}} \quad \text{and} \quad \lambda = \frac{c}{f}$$

into Equation (1b) and after a bit of algebra, we find

$$f^{'} = \frac{f m_e c}{hf(1 - \cos\theta) + m_e c} \qquad (1c)$$

By substituting Equation (1c) into 1(a) and after a bit algebra, the kinetic energy of the recoiling electrons given by

$$KE_{recoil\ electron} = \frac{2\left(\frac{h}{m_e c}\right)\sin^2\left(\frac{\theta}{2}\right)}{\frac{c}{f} + 2\left(\frac{h}{m_e c}\right)\sin^2\left(\frac{\theta}{2}\right)} hf \qquad (1d)$$

(c) From Equation (1d), the kinetic energy of the recoiling electron become maximum when $\sin^2\left(\frac{\theta}{2}\right)$ = 1. Consequently,

$$KE_{max}(recoiling\ electron) = \frac{2\left(\frac{h}{m_e c}\right)}{\frac{c}{f} + 2\left(\frac{h}{m_e c}\right)} hf \qquad (1e)$$

After a bit algebra, the maximum kinetic energy of the recoiling electron is

$$KE_{max}(recoiling\ electron) = hf \frac{\frac{2hf}{m_e c^2}}{1 + \frac{2hf}{m_e c^2}} \qquad (1f)$$

(c) (i)

From (c), the kinetic energy of the recoiling electron become maximum when

$\sin^2\left(\frac{\theta}{2}\right) = 1$ or $\theta = 0°$, 180° and 360°

Therefore, the kinetic energy of the recoiling electron become maximum when

$\theta = 180°$ (head-on collision) because the shift in wavelength is zero at 0°.

and $\phi = 0°$.

(c) (ii)

Here, $\phi = 0°$, $\theta = 180°$, and KE_{max} (*recoiling electron*) = 100 keV.
By substituting these values into Equation 1(d) and solving the equation for *hf*, we obtain

$$hf = 0.2173 \text{ MeV}$$

[*Hint*: In order to get this answer, you need to get the quadratic equation and then solves the equation by using:

$$x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a} .$$
]

Finally, replace the hf = 0.2173 MeV and KE_{max} (*recoiling electron*) = 100 keV into Equation (1a), we obtain

$$\text{Energy of scattered photon } = hf'$$
$$= hf - KE_{recoil\ electron}$$
$$¿(0.2173 - 0.1) \text{ MeV}$$
$$¿0.173 \text{ MeV}$$

2. (i) $$\Delta t \sim \frac{\hbar}{2\Delta E} = \frac{\hbar}{2mc^2}$$

(ii) $$x = c\,\Delta t \sim \frac{\hbar}{2mc}$$

(iii) take $x = r$,

$$r \sim \frac{\hbar}{2mc} \Rightarrow mc^2 \sim \frac{\hbar c}{2r} = \frac{hc}{4\pi r} = \frac{1240 \text{ eV} \cdot 10^{-9} m}{4\pi \times 10^{-15} m} = \frac{1240 \text{ eV} \cdot 10^{-9} m}{4\pi \times 10^{-15} m} = 98.7 \text{MeV} \sim 100 \text{MeV}$$

(iv)

$$\Delta t \sim \frac{\hbar}{2\Delta E} = \frac{\hbar c}{2cmc^2} = \frac{hc}{4\pi cmc^2} = \frac{1240 \text{ eV} \cdot 10^{-9} m}{4\pi \times (3 \times 10^8 \text{m/s}) 100 \text{MeV}} = \frac{1240 \text{ eV} \cdot 10^{-9}}{4\pi \times (3 \times 10^8)(100 \times 10^6) \text{eV}} s$$
$$¿\frac{1240\, 10^{-9}}{4\pi \times (3 \times 10^8)(100 \times 10^6)} s \sim 10^{-24} s$$