ZCT 104/3
Modern Physics
Semester II, Academic Year 2008/2009
School of Physics, Universiti Sains Malaysia
Test – 27 Mac 2009
Duration: 1 hour

Name: ______________________ Matric No.: ______________________

Class: A / B Tutorial Group: ______________________

Answer all questions.

1. A proton is accelerated from rest to a velocity of 0.95*c*. Calculate the energy needed. Given that the rest energy of the proton is 938 MeV.

[**15 marks**]

Answer:

- Find the total energy of the proton for each sate:

 Initial state:

$$E_i = \text{Kinetic energy} + \text{rest energy}$$
$$= 0 + m_p c^2$$

 Final state:

$$E_f = \text{Kinetic energy} + \text{rest energy} = mc^2$$
$$= \gamma m_p c^2 = m_p c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = m_p c^2 \left(\frac{1}{\sqrt{1 - \frac{(0.95c)^2}{c^2}}} \right) = 3.2 m_p c^2$$

- Consequently, the change in energy is:

$$\text{Change in energy} = E_{final} - E_{initial}$$
$$= 3.2 m_p c^2 - m_p c^2$$
$$= 2.2 m_p c^2$$
$$= 2.2(938\ \text{MeV})$$
$$= 2.064 \times 10^3\ \text{MeV}$$

- Hence, the energy needed to accelerate a proton initially at rest to a velocity of 0.95*c* is:

 Energy needed = change in energy

 = 2.064×10^3 MeV

2. A 100 keV photon scatters from a free electron initially at rest. Find the recoil velocity (magnitude only) of the electron if the photon scattering angle is 180°. Given that the rest energy of the electron is 0.511 MeV.

[**35 marks**]

Answer:

- Here, E = 100 keV, θ = 180°, and $\Delta\lambda = 2\times\lambda_c = 2(0.00243\text{ nm}) = 0.00486\text{ nm}$.
- The λ of the incident photon is:

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{100\,\text{keV}} = 0.0124\,\text{nm}$$

- Consequently, the λ' of the scattered photon is:

$$\lambda' - \lambda = \Delta\lambda$$

$$\lambda' = \Delta\lambda + \lambda = 0.00486\,\text{nm} + 0.0124\,\text{nm} = 0.01726\,\text{nm}$$

- Hence, the energy of the scattered photon is:

$$E' = \frac{hc}{\lambda'} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.01726\,\text{nm}} = 71.84\,\text{keV}$$

- From the conservation of energy,

$$E_{initial} = E_{final}$$

$$E = E' + K'_e$$

$$K'_e = E - E' = 100\,\text{keV} - 71.84\,\text{keV} = 28.16\,\text{keV}$$

- The total relativistic energy of the recoiled electron is:

$$\begin{aligned} E &= mc^2 \\ &= K'_e + m_e c^2 \\ &= 28.16\,\text{keV} + 0.511\,\text{MeV} \\ &= 0.5392\,\text{MeV} \end{aligned}$$

- Since $E = mc^2 = \gamma m_e c^2$,

$$\therefore \quad \gamma = \frac{0.5392\,\text{MeV}}{0.511\,\text{MeV}} = 1.055$$

- The velocity of the recoiled electron can be obtained by:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.055$$

$$v = 0.319\,c$$

3. Say you have experimentally measured all the wavelengths in the emission spectral lines from the Hydrogen atom, λ_α, λ_β, λ_γ ,… λ_∞. Describe clearly how to use these measured wavelengths to extract the value of the Rydberg constant R_H as given in the Balmer's empirical formula $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$.

Note: This question does not ask you what the value of R_H is. Instead you are asked to provide a description of how to manipulate the experimental data of the measured values of λ_α, λ_β, λ_γ ,… λ_∞ so that one can arrive at the experimental value of R_H. No marks will be given if you simply give the value of R_H.

[50 marks]

Answer:

Experimental measurement of the Rydberg constant, R_H

One measures the wavelengths of the α, β, γ, …lines (corresponding to $n = 3, 4, 5, \ldots\infty$) in Balmer's empirical formula $\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$
Then plot $1/\lambda$ as a function of $1/n^2$. Note that here $n \geq 3$.