

UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

ZCT 104/3 - Physics IV (Modern Physics)
[Fizik IV (Fizik Moden)]Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **TWENTY TWO** printed pages before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH DUA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instruction: Answer **ALL** the objective questions in **Section A**. Answer only **SIX** questions out of **EIGHT** questions given in **Section B**.

Students are allowed to answer all questions in Bahasa Malaysia or in English.

[Arahan: Jawab **KESEMUA** soalan objektif dalam **Bahagian A**. Jawab **ENAM** soalan daripada **LAPAN** soalan yang disediakan dalam **Bahagian B**.

[Pelajar dibenarkan untuk menjawab samada dalam bahasa Malaysia atau bahasa Inggeris.]

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Data

Speed of light in free space, $c = 3.00 \times 10^8 \text{ m s}^{-1}$
 Permeability of free space, $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
 Permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
 Elementary charge, $e = 1.60 \times 10^{-19} \text{ C}$
 Planck constant, $h = 6.63 \times 10^{-34} \text{ J s}$
 Rydberg constant, $R_H = 1.097 \times 10^7 \text{ m}^{-1}$
 Unified atomic mass constant, $u = 1.66 \times 10^{-27} \text{ kg}$
 Rest mass of electron, $m_e = 9.11 \times 10^{-31} \text{ kg}$
 Rest mass of proton, $m_p = 1.67 \times 10^{-27} \text{ kg}$
 Molar gas constant, $R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
 Avogadro constant, $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
 Gravitational constant, $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
 Acceleration of free fall, $g = 9.81 \text{ m s}^{-2}$

Section A: Objective questions. (40 marks)

[Bahagian A: Soalan-soalan objektif (40 markah)]

Instruction: Answer all the objective questions in the OMR form provided.

[Arahan: Jawab kesemua soalan objektif dalam borang OMR yang dibekalkan.]

- In the following, which statement(s) is (are) correct regarding two events, A and B, at the space-time coordinates (x_A, t_A) and (x_B, t_B) ? Assume $x_A \neq x_B$, $t_A \neq t_B$.
 [Kenyataan-kenyataan berikut yang manakah adalah benar berkenaan dengan dua peristiwa A dan B, pada koordinat ruang-masa (x_A, t_A) dan (x_B, t_B) ? Anggap $x_A \neq x_B$, $t_A \neq t_B$.]
 - The time interval between events A and B as measured by an observer O_1 , in general, is different from that measured by another observer O_2 , at relative motion with respect to O_1 .
 [Selang masa di antara peristiwa A dan B yang diukur oleh seorang pemerhati O_1 , secara amnya, adalah berbeza daripada selang masa yang diukur oleh pemerhati lain, O_2 , yang bergerak secara relatif kepada O_1 .]

11. Which of the following statements is (are) true regarding an electron-positron pair annihilation process?

[Yang manakah kenyataan(-kenyataan) berikut adalah benar berkenaan dengan proses habis-binasa elektron-positron?]

I. The electron's linear momentum must be equal in magnitude but opposite to that of the positron.

[Momentum elektron mestilah sama magnitud tapi berlawanan arah dengan momentum linear positron.]

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II. The resultant product can be a neutrino-antineutrino pair.

[Hasil produk boleh jadi suatu pasangan neutrino-antineutrino.]

III. The resultant product can be a proton-antiproton pair.

[Hasil produk boleh jadi suatu pasangan proton-antiproton.]

IV. The resultant product can consist of only a single photon.

[Hasil produk boleh jadi hanya mengandungi foton yang tunggal.]

A. II, III
III

B. I, II, IV

C. I, II,

D. I, III, IV

E. None of A, B, C, D

12. In a typical Compton scattering experiment,

[Dalam suatu eksperimen serakan Compton yang tipikal,]

I. the outgoing X-ray photon will increase in its energy in the forward direction.

[foton sinar-X yang keluar akan bertambah dalam tenaga dalam arah ke hadapan.]

II. the outgoing X-ray photon will increase in the magnitude of the momentum in the forward direction.

[foton sinar-X yang keluar akan bertambah dalam magnitud momentum dalam arah ke hadapan.]

III. The Compton electron will increase in its rest mass.

[Elektron Compton akan bertambah dalam jisim rehatnya.]

IV. the Compton electron will increase in total energy.

[Elektron Compton akan bertambah dalam jumlah tenaganya.]

A. III, IV

B. I, II

C. I, III

D. IV

E. None of A, B, C, D

13. Due to the Heisenberg uncertainty principle, a particle called meson is thought to be created spontaneously within a nucleus of a typical size $r \sim \text{fm}$. Assume that the meson travels at speed $\sim c$ within the nucleus, what is the estimate of the mass of meson?

[Disebabkan oleh prinsip ketidakpastian Heisenberg, suatu zarah dikenali meson difikirkan tercipta secara spontan dalam nukleus yang bersaiz $r \sim \text{fm}$. Anggap bahawa meson bergerak pada laju $\sim c$ dalam nukleus. Apakah anggaran bagi jisim meson?]

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A. 0

B. $\frac{r}{hc}$

C. $\frac{\hbar c}{r}$

D. $\frac{rc}{\hbar}$

E. $\frac{\hbar}{rc}$

14. Consider a matter particle with rest mass m_0 , moving with a speed v . Which of the following statements is (are) true regarding its de Broglie wave?

[Pertimbangkan suatu zarah jirim berjisim rehat m_0 bergerak dengan laju v . Yang manakah kenyataan(-kenyataan) berikut adalah benar berkenaan dengan gelombang de Broglieya]

I. The de Broglie wavelength of the matter particle is a constant and does not depend on the speed v .

[Jarak gelombang de Broglie zarah jirim adalah suatu pemalar dan tidak bergantung kepada v .]

III. The envelope wave generally has longer wavelength than the phase wave.
 [Gelombang sampul secara amnya mempunyai jarak gelombang yang lebih panjang berbanding dengan gelombang fasa.]

IV. The speed of the phase wave could possibly be faster than the speed of light, c .
 [Laju gelombang fasa mungkin lebih besar daripada laju cahaya, c .]

- | | | |
|-------------------|-----------------------|------------|
| A. I, II, III, IV | B. I, II, III | C. II, III |
| D. I, III | E. None of A, B, C, D | |

17. Which of the following is (are) true while deriving the Rayleigh-Jeans formula for blackbody radiation?

[Yang manakah kenyataan(-kenyataan) berikut adalah benar semasa menerbitkan rumus Rayleigh-Jeans untuk pancaran jasad hitam?]

I. Electromagnetic radiation is at thermodynamical equilibrium with the cavity wall of the black body.

[Pancaran elektromagnet berada pada keadaan keseimbangan termodinamik dengan dinding kaviti jasad hitam.]

II. Average energy of each mode of the standing electromagnetic wave in the cavity is frequency dependent.

[Tenaga purata setiap mod pegun gelombang elektromagnet dalam kaviti tersandar pada frekuensi.]

III. Average energy of each mode of the standing electromagnetic wave in the cavity is proportional to temperature.

[Tenaga purata setiap mod pegun gelombang elektromagnet dalam kaviti berkadaran kepada frekuensi.]

IV. The temperature of the black body is constant.

[Suhu jasad hitam adalah malar.]

- | | | |
|-------------------|-----------------------|------------|
| A. I, II, III, IV | B. I, II, IV | C. I, III, |
| D. II, III | E. None of A, B, C, D | |

IV

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18. Which of the following is (are) true while deriving the Planck formula for blackbody radiation?

[Yang manakah kenyataan(-kenyataan) berikut adalah benar semasa menerbitkan rumus Planck untuk pancaran jasad hitam?]

I. Electromagnetic radiation is at thermodynamical equilibrium with the cavity wall of the black body.

[Pancaran elektromagnet berada pada keadaan keseimbangan termodinamik dengan dinding kaviti jasad hitam.]

II. Average energy of each mode of the standing electromagnetic wave in the cavity is frequency dependent.

[Tenaga purata setiap mod pegun gelombang elektromagnet dalam kaviti tersandar pada frekuensi.]

III. Average energy of each mode of the standing electromagnetic wave in the cavity is proportional to temperature.

[Tenaga purata setiap mod pegun gelombang elektromagnet dalam kaviti berkadaran kepada frekuensi.]

IV. The temperature of the black body is constant.

[Suhu jasad hitam adalah malar.]

- | | | |
|-------------------|-----------------------|------------|
| A. I, II, III, IV | B. I, II, IV | C. I, III, |
| D. II, III | E. None of A, B, C, D | |

IV

19. Which of the following statements is (are) true regarding the relative probabilities of photon absorption in a metal target?

[Yang manakah kenyataan(-kenyataan) berikut adalah benar mengenai kebarangkalian relatif penyerapan foton dalam suatu sasaran logam?]

- I. Typically, a x-ray photon at a given energy has larger probability to undergo Compton scattering in metal with higher atomic number than that with lower atomic number.
 [Secara tipikal, suatu foton sinar-X pada tenaga tertentu mempunyai kebarangkalian lebih tinggi untuk menjalani serakan Compton dalam logam dengan nombor atom yang lebih besar berbanding dengan logam bernombor atom yang lebih rendah.]
- II. Typically, an infrared photon at a given energy has larger probability to undergo Compton scattering than photoelectric effect in a given metal target.
 [Secara tipikal, suatu foton infra merah pada tenaga tertentu mempunyai kebarangkalian lebih tinggi untuk menjalani serakan Compton berbanding dengan kesan fotoelektrik dalam suatu logam yang tertentu.]

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- III. Typically, a gamma-ray photon at a given energy has larger probability to undergo Compton scattering than photoelectric effect in a given metal target.
 [Secara tipikal, suatu foton sinar gama pada tenaga tertentu mempunyai kebarangkalian lebih tinggi untuk menjalani serakan Compton berbanding dengan kesan fotoelektrik dalam suatu logam yang tertentu.]
- IV. Typically, a sufficiently energetic gamma-ray photon has larger probability to undergo creation of electron-positron pair in target with larger atomic number than that with smaller atomic number.
 [Secara tipikal, suatu foton yang bercukup tenaga mempunyai kebarangkalian lebih tinggi untuk menjalani penghasilan pasangan elektron-positron dalam logam dengan nombor atom yang lebih besar berbanding dengan logam bernombor atom yang lebih rendah.]

- A. I, II, III B. I, II, IV C. I, III, IV
- D. III, IV E. None of A, B, C, D

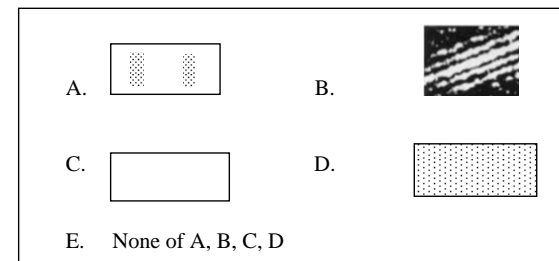
20. An electron has a de Broglie wavelength λ . In general, its kinetic energy is [Suatu elektron bergerak gelombang de Broglie λ . Secara amnya, tenaga kinetiknya adalah]

- A. less than or equal to $\frac{h^2}{2\lambda^2 m_e}$ [kurang atau sama dengan $\frac{h^2}{2\lambda^2 m_e}$]
- B. larger than or equal to $\frac{h^2}{2\lambda^2 m_e}$ [lebih besar atau sama dengan $\frac{h^2}{2\lambda^2 m_e}$]
- C. strictly equals to $\frac{h^2}{2\lambda^2 m_e}$ [secara tepatnya bersamaan dengan $\frac{h^2}{2\lambda^2 m_e}$]
- D. not related to λ [tidak berkaitan dengan λ]
- E. None of A, B, C, D

...14/-

- 14 -

For questions 21 – 24, choose the answer from the pattern as depicted in the following choices:
 [Untuk soalan-soalan 21 – 24, pilih jawapan daripada corak yang dipaparkan dalam pilihan yang berikutnya]



21. Consider a Young double slit experiment with electron being fired from the electron gun once in a time, as depicted in Figure 1.
 [Pertimbangkan suatu eksperimen dwi-celah Young dengan elektron ditembakkan daripada senapang elektron satu setiap kali, sebagaimana yang ditunjukkan dalam Gambarajah 1.]

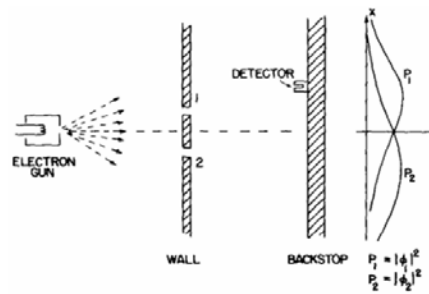


Figure 1 [Gambarajah 1]

What is the distribution pattern of the detected electron?
 [Apakah corak taburan elektron yang dikesan?]

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22. Consider the Young double slit experiment in Question 21 again, but this time with the double slits equipped with conducting coil such that we can gain the information of which slit an electron is pass through each time. See Figure 2.

[Pertimbangkan eksperimen dwi-celah Young dalam soalan 21, tapi kali ini dwi-celah itu dilengkapi dengan lingkaran konduksi supaya kita boleh memperoleh maklumat celah mana yang elektron lalu setiap kali. Rujuk Gambarajah 2.]

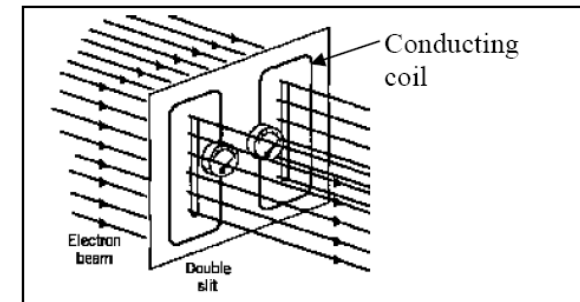


Figure 2 [Gambarajah 2]

What is the distribution pattern of the detected electron?
 [Apakah corak taburan elektron yang dikesan?]

23. Consider the Young double slit experiment in Question 21 again, but this time with the electron replaced by photon that is fired one in a time. What is the distribution pattern of the detected electron?
 [Pertimbangkan eksperimen dwi-celah Young dalam soalan 21, tapi kali ini elektron digantikan dengan foton yang ditembak satu setiap kali. Apakah corak taburan elektron yang dikesan?]
24. Following question 23, with the experimental set up of Figure 2, which distribution pattern represents the detected photon?
 [Susulan daripada soalan 23, dengan setup eksperimen seperti dalam Gambarajah 2, apakah corak taburan foton yang dikesan?]

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25. Consider a photoelectric effect experiment. Which of the following statements is (are) true?
 [Pertimbangkan suatu eksperimen kesan fotoelektrik. Yang manakah kenyataan(kenyataan) berikut adalah benar?]

- I. The measured stopping potential carries the information of the maximum kinetic energy of the photoelectron.
 [Keupayaan penghenti yang terukur membawa maklumat tentang tenaga kinetik maksima fotoelektron.]
- II. The saturated photo current in the I - V characteristic curve reflects the intensity of the incident light.
 [Arus tepu foto dalam lengkungan cirian I - V memantulkan keamatan cahaya tuju.]
- III. The kinetic energy of the photoelectron depends on the intensity of the incident light.
 [Tenaga kinetik fotoelektron bersandar kepada keamatan cahaya tuju.]
- IV. The cut-off frequency of a given metal depends on the frequency of the incident light used in the experiment.
 [Frekuensi ambang bagi sesuatu logam bersandar kepada frekuensi cahaya tuju yang digunakan dalam eksperimen.]

- IV A. I, II, III, IV B. I, II C. I, III,
 D. II, III E. None of A, B, C, D

Section B: Structural questions. (60 marks)

[Bahagian B: Soalan-soalan struktur.(60 markah)]

Instruction: Answer **SIX** questions out of eight questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

[Arahan: Jawab **ENAM** soalan daripada lapan soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]

1. [a] Particle A is moving with a constant velocity $+0.90c$ relative to an Earth observer. Particle B moves in the opposite direction with a constant velocity $-0.90c$ relative to the same Earth observer. Determine the velocity of particle B as seen by particle A by using
 [Zarah A bergerak dengan halaju malar $+0.90c$ relatif kepada seorang pemerhati di Bumi. Zarah B bergerak dalam arah yang bertentangan dengan halaju malar $-0.90c$ relatif kepada pemerhati yang sama di Bumi. Tentukan halaju zarah B seperti yang diperhatikan oleh A dengan menggunakan]
- [i] Galilean approach.
 [pendekatan Galileo.]
- [ii] Special relativity approach.
 [pendekatan teori kerelatifan khas.]

Please state clearly all the symbols used in your working.
 [Sila nyatakan dengan jelas simbol-simbol yang digunakan dalam kerja anda.]

- [b] A square object of area 100 cm^2 is at rest in the reference frame of O . If O' is moving at a velocity $0.8c$ relative to O and in the direction along a diagonal of

the square,

[*Satu objek segiempat sama yang berluas 100 cm^2 berada pada keadaan rehat dalam rangka rujukan O . Jika O' bergerak dengan halaju $0.8c$ relatif kepada O dan dalam arah sepanjang pepenjuru segiempat sama itu,*]

- [i] sketch the shape of the object as observed by O' , and
[*lakarkan bentuk objek itu seperti yang diperhatikan oleh O' , dan*]
- [ii] determine the area of the object as measured by O' .
[*tentukan luas objek itu seperti yang diukur oleh O' .*]

[10/100]

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2. [a] For an observer P, two events at x_1 and $x_1 + 600 \text{ km}$ are simultaneously. What is the time interval between these events according to an observer P' which found that the space interval between these events is 1200 km ?
[*Bagi seorang pemerhati P, dua peristiwa di x_1 dan $x_1 + 600 \text{ km}$ adalah serentak. Berapakah selang masa antara dua peristiwa itu menurut seorang pemerhati P' yang mendapati bahawa selang ruang antara dua peristiwa itu ialah 1200 km ?*]

- [b] Calculate the energy needed to accelerate an electron from $v = 0.8c$ to $v = 0.99c$.
[*Hitungkan tenaga yang diperlukan untuk memcutkan satu elektron daripada $v = 0.8c$ ke $v = 0.99c$.*]

- [c] What are the essential modification of classical theory did Planck make in dealing with the problem of black body radiation?
[*Apakah pengubahsuaian asas dalam teori klasik yang telah Planck lakukan semasa mengendalikan masalah sinaran jasad hitam?*]

[10/100]

3. [a] A photon of wavelength 331 nm is incident on photocathode and ejects an electron of energy $3 \times 10^{-19} \text{ J}$. If the wavelength of the incident photon is changed to 500 nm , the energy of the ejected electron is $0.972 \times 10^{-19} \text{ J}$. Determine
[*Satu foton berjarak gelombang 331 nm ditujukan ke atas fotokatod dan menyingkirkan satu elektron yang bertenaga $3 \times 10^{-19} \text{ J}$. Jika jarak gelombang foton tuju itu berubah kepada 500 nm , tenaga elektron tersingkir ialah $0.972 \times 10^{-19} \text{ J}$. Tentukan]*

- [i] the value of Planck's constant,
[*nilai pemalar Planck,*]
- [ii] the threshold wavelength, and
[*jarak gelombang ambang, dan*]
- [iii] the work function for the photocathode.
[*fungsi kerja bagi fotokatod itu.*]

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- [b] A 0.700 MeV photon scatters off a free electron initially at rest such that the scattering angle of the scattered photon is twice the scattering angle of the scattered electron, as shown in Figure 3. Determine
[*Satu foton 0.700 MeV menyerakkan satu elektron bebas yang asalnya pada keadaan rehat dengan sudut serakan bagi foton terserak ialah dua kali ganda daripada sudut serakan bagi elektron yang terserak, iaitu seperti yang ditunjukkan dalam Gambarajah 3. Tentukan]*

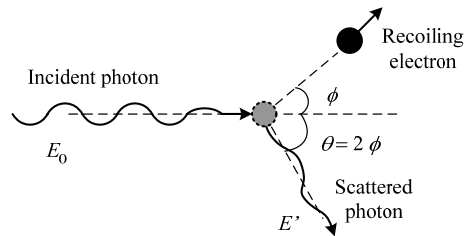


Figure 1.0 3

- [i] the scattering angle for the electron, and
[sudut serakan bagi elektron, dan]
- [ii] the final speed of the electron.
[laju akhir bagi elektron.]

[10/100]

4. [a] Show that the ratio of the Compton wavelength $\lambda_c = h/(m_e c)$ to the de Broglie wavelength $\lambda = h/p$ for a relativistic electron is
[Tunjukkan bahawa nisbah jarak gelombang Compton $\lambda_c = h/(m_e c)$ kepada jarak gelombang de Broglie $\lambda = h/p$ bagi satu elektron relativistik adalah]

$$\frac{\lambda_c}{\lambda} = \sqrt{\left(\frac{E}{m_e c^2}\right)^2 - 1}$$

where E is the total energy of the electron and m_e is the rest mass of the electron.
[di mana E jumlah tenaga bagi elektron dan m_e ialah jisim rehat bagi elektron.]

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- [b] The resolving power of a microscope depends on the wavelength used. If one wished to “see” an atom, a resolution of approximately 1.00×10^{-11} m would be required.

[Kuasa pelepasan bagi suatu mikroskop bergantung kepada jarak gelombang yang digunakan. Jika ingin untuk “melihat” satu atom, suatu pelepasan yang berhampiran 1.00×10^{-11} m adalah diperlukan.]

- [i] If electrons are used (in an electron microscope), what minimum kinetic energy is required for the electrons?
[Jika elektron digunakan (dalam suatu mikroskop elektron), apakah tenaga kinetik minimum yang diperlukan untuk elektron?]
- [ii] If photons are used, what minimum photon energy is needed to obtain the required resolution?
[Jika foton digunakan, apakah tenaga foton minimum yang dikehendaki untuk memperoleh pelepasan yang diperlukan?]

[10/100]

5. [a] Given that a hydrogen atom emits a photon of wavelength 410.1 nm, identify the initial and the final states of the atom.
[Diberi bahawa satu atom hidrogen mengeluarkan satu foton berjarak gelombang 410.1 nm, tentukan keadaan awal dan keadaan akhir bagi atom itu.]

- [b] Suppose that in a Franck-Hertz experiment, electrons of energy 13.0 eV is used to excite hydrogen atoms.
[Andaikan bahawa dalam suatu eksperimen Franck-Hertz, elektron-elektron yang bertenaga 13.0 eV telah digunakan untuk mengujakan atom-atom hidrogen.]

- [i] What is the maximum amount of energy the incident electron can lose inelastically in this collision?
[Apakah jumlah tenaga maksimum elektron tuju itu dapat hilang secara tak kenyal dalam perlanggaran ini?]
- [ii] What is the remaining energy of the electron?
[Apakah tenaga baki bagi elektron itu?]
- [iii] What spectral lines will the hydrogen atoms emit under these

conditions?

[Apakah garis-garis spektrum yang akan dihasilkan oleh atom-atom hidrogen di bawah keadaan ini?]

[10/100]

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6. Consider the Bohr's hydrogen model.
[Pertimbangkan model hidrogen Bohr.]

[a] Consider only mechanical stability and classical physics, show that the kinetic energy, K , and potential energy, U , for the single electron in a circular orbit is related via $U = -2K$. Please explain your steps clearly and define all the symbols used.

[Pertimbangkan hanya kestabilan mekanik dan fizik klasik, tunjukkan bahawa tenaga kinetik, K , dan tenaga keupayaan, U , bagi suatu elektron tunggal dalam orbit bulatan adalah dikaitkan oleh $U = -2K$. Sila terangkan langkah-langkah anda, dan jelaskan semua simbol yang digunakan.]

[b] By incorporating the quantisation condition on the angular momentum $L = n\hbar$, $n=1, 2, 3, \dots$, derive the allowed values for the electron's orbit, r_n .
[Dengan mengambilkira syarat pengkuantuman dalam momentum sudut $L = n\hbar$, $n=1, 2, 3$, terbitkan nilai-nilai yang dibenarkan bagi orbit elektron, r_n .]

[c] What is the numerical value for the Bohr's radius in nm?
[Apakah nilai berangka bagi jejari Bohr dalam nm?]

[10/100]

7. Consider a proton and an electron, both moving at a common kinetic energy, K . Assuming relativistic scenario,
[Pertimbangkan suatu proton dan satu elektron, kedua-duanya bergerak pada suatu tenaga kinetik, K yang sama. Anggapkan keadaan kerelatifan,]

[a] derive the ratio of λ_e/λ_p .
[terbitkan nisbah λ_e/λ_p .]

[b] What is the ratio in (a) for nonrelativistic scenario?
[Apakah nisbah dalam (a) bagi keadaan tidak kerelatifan?]

λ refers to the de Broglie wavelength of these particles.
[λ merujuk kepada jarak gelombang de Broglie zarah-zarah tersebut.]

[10/100]

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8. A particle of mass m is placed in a one-dimensional box of length L . The box is so small that the particle's motion is relativistic, so that the kinetic energy expression $K = p^2/2m$ is not valid.

[Suatu zarah berjisim m diletakkan dalam suatu kotak satu dimensi dengan panjangnya L . Kotak itu adalah cukup kecil sehinggakan gerakan zarah adalah relativistik, supaya ungkapan tenaga kinetik $K = p^2/2m$ adalah tidak terpakai.]

[a] Derive an expression for the energy levels of the particle using the relativistic energy- momentum relation and the quantization of momentum that derives from confinement.

[Terbitkan suatu ungkapan untuk paras-paras tenaga zarah dengan menggunakan hubungan relativistik tenaga-momentum dan pengkuantuman momentum yang diterbitkan daripada pengurungan.]

[b] If the particle is an electron in a box of length $L = \frac{h}{2mc}$, find its lowest possible energy. m refers to the rest mass of electron.

[Jika zarah tersebut adalah elektron dalam kotak dengan panjang $L = \frac{h}{2mc}$, dapatkan tenaganya yang paling rendah. m merujuk kepada jisim rehat elektron.]

- [c] By what percent is the nonrelativistic formula for the energy in error?
 [Dalam sebutan peratusan, sejauh manakah rumus tenaga tidak kerelatifan itu salah?]

[10/100]

- 000 O 000 -

Solution to ZCT 104 final exam, semester II, academic session 0809.**Section A (objectives)**

1.A (I, III are true)

2.B. (II, IV are true)

3.A, $\sim 10^{(-19)}$.

$p = E/c = 1\text{GeV}/c \sim 10^9 \times (1.6 \times 10^{(-19)}) / (3 \times 10^8) = (1.6/3) \times 10^{(9-19-8)} \sim 10^{(-18)}$. (choose the closest order, -19)

$$4D, \frac{c\alpha}{\sqrt{1+\alpha^2}}$$

Time taken to travel via a distance of L is $t = (L/\gamma)/v < \tau$. Solve for v gives $\frac{c\alpha}{\sqrt{1+\alpha^2}}$.

5B (II, IV are true)

6A (I, III are true)

7E (I, II, IV are true)

8E (I, III, IV are true)

9A (I, III are true)

10C (I, II, III are true)

11.A (II, III are true)

12. D (IV only is true)

13.E

14. D (only III is true). $\lambda = \lambda(m_0, v) = \frac{h}{\gamma m_0 v} < \frac{h}{m_0 v}$.

15. C (only I is true)

16. A (all true)

17. C (I, III, IV are true).

18. B (I, II, IV are true).

19. D (III, IV are true)

20. B. $K_{sr} \geq K_{classical} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

21. B
 22. A
 23. B
 24. B (photon unaffected by coil detection).
 25. B. (I, II are true).

Section B (structured questions)

Q1[a] [Physics, Giambattista, 2008, p. 979, Q=26, modified]
 Q1[b] [Physics, Giambattista, 2008, p. 979, Q=26, modified]

Q2[a] [Teori Kerelatifan Khas – Liew Yong Choy, p.71]
 Q2[b] [Teori Kerelatifan Khas – Liew Yong Choy, p.94]

Q3[a] (Chand, Q =13, p.167)
 Q3[b] [Serway, 6th edition, p. 1316, Q= 28.]
 Q4[a] Serway, 6th edition, p. 1319, Q= 65.]
 Q4[b] [Serway, 6th edition, p. 1317, Q= 41.]

Q5[a] [Ohanian, p. 137, Q= 25, modified.]

[1] [a]

Answer:

- Here O' is the particle A; O' moves with a speed of $v = +0.9c$ relative to O (Earth). The particle B moves at speed $u_x = -0.90c$ relative to O.
- Hence, we need to find the speed of B, v , relative to particle A (O').

[i]

- By using the Galilean approach, the speed of B, v , relative to particle A (O') is:
 $u_x' = u_x - v = -0.9c - (0.90c) = -1.8 c$

[ii]

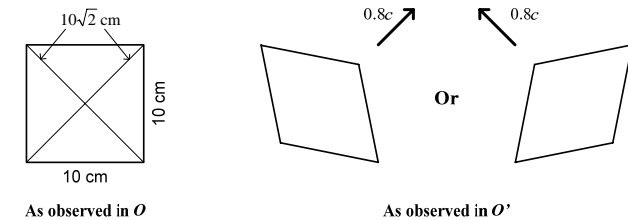
- By using the Special relativity approach, the speed of B, v , relative to particle A (O') is:

$$u_x' = \frac{u_x - v}{1 - \left(\frac{v}{c}\right)\left(\frac{u_x}{c}\right)} = \frac{-0.9c - 0.9c}{1 - \left[\frac{0.9c}{c}\right]\left(-0.9c\right)} = \frac{-1.8c}{1.81} = -0.994c$$

[1] [b]

Answer:

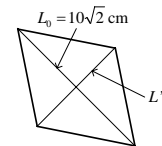
[i]



[ii]

- The proper length, $L_0 = 10\sqrt{2} = 14.142$ cm.
- Length contraction only happens along the direction of motion.
- The observer sees the diagonal length of the square to be contracted to a length of

$$L' = (l/\gamma)L_0 = 14.142 \left(\sqrt{1 - \frac{(0.8c)^2}{c^2}} \right) = 14.142(0.6) = 8.485 \text{ cm}$$



Where
$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = 1.667$$

- The area of the object as measured by O' is

$$A' = \frac{1}{2} \times L_0 \times L' = \frac{1}{2} \times (10\sqrt{2}) \times 8.485 = 59.997 \text{ cm}^2 \approx 60 \text{ cm}^2$$

[2] [a]

Answer:

- Here, $\Delta t = 0 \text{ s}$, $\Delta x = 600 \text{ m}$; $\Delta x' = 1200 \text{ m}$, $\Delta t' = ?$
- The time interval between these events according to an observer P' is given by:

$$\Delta t' = \frac{(\Delta x')(v/c^2)}{\sqrt{1 - (v^2/c^2)}} \quad (1)$$

- In order to solve Eq. (1), we need to find out the speed, v , of the P' .
- Use the length contraction formula. Here, the proper length, $L_0 = 1200 \text{ km}$ and the improper length, $L = 600 \text{ km}$:

$$L = (1/\gamma)L_0 = L_0 \left(\sqrt{1 - \frac{v^2}{c^2}} \right)$$

$$600 = 1200 \times \left(\sqrt{1 - \frac{v^2}{c^2}} \right) \quad (2)$$

$$v = 0.866c$$

- Consequently, the time interval between these events according to an observer P' is:

$$\begin{aligned} \Delta t' &= \frac{(1200 \text{ km})(0.866c/c^2)}{\sqrt{1 - \left[\frac{(0.866c)^2}{c^2} \right]}} = \frac{(1200 \times 10^3)(0.866c/c^2)}{\sqrt{1 - \left[\frac{(0.866c)^2}{c^2} \right]}} \\ &= \frac{1.0392 \times 10^6 / c}{0.5} = 6.928 \times 10^{-3} \text{ s} \end{aligned}$$

- This is not zero. This indicates that two events at x_1 and $x_1 + 600 \text{ km}$, which are simultaneous to the observer in P , do not appear so to the observer in P' .

[2] [b]

Answer:

- Find the total energy of the electron for each state:

Initial state:

$$E_i = \text{Kinetic energy} + \text{rest energy} = mc^2$$

$$= \gamma m_e c^2 = m_e c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = m_e c^2 \left(\frac{1}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} \right) = \frac{1}{0.60} m_e c^2 = 1.667 m_e c^2$$

Final state:

$$E_f = \text{Kinetic energy} + \text{rest energy} = mc^2$$

$$= \gamma m_e c^2 = m_e c^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = m_e c^2 \left(\frac{1}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}} \right) = \frac{1}{0.141} m_e c^2 = 50.251 m_e c^2$$

- Consequently, the change in energy is:

$$\begin{aligned} \text{Change in energy} &= E_{\text{final}} - E_{\text{initial}} \\ &= 50.251 m_e c^2 - 1.667 m_e c^2 \\ &= 48.584 m_e c^2 \\ &= 48.584 (0.511 \text{ MeV}) \\ &= 24.826 \text{ MeV} \end{aligned}$$

- Hence, the energy needed to accelerate a proton initially at rest to a velocity of $0.95c$ is:

$$\begin{aligned} \text{Energy needed} &= \text{change in energy} \\ &= 24.826 \text{ MeV} \end{aligned}$$

[2] [c]

Answer:

- The essential modification of classical theory did Planck make in dealing with the problem of black body radiation is:

The oscillators **can only have certain discrete energies** determined by:

$$E_n = nhf$$

where **n is an integer (0, 1, 2, 3, ...)**, f is the frequency, and h is called Planck' constant.

- As a consequence, The oscillators can **absorb** or **emit energy in discrete multiples** of the fundamental quantum of energy given by:

$$\Delta E = hf$$

[3] [a]

Answer:

- Given that:
 - When $\lambda_1 = 331$ nm, the kinetic energy of the photoelectron is $K_1 = 3 \times 10^{-19}$ J or 1.875 eV.
 - When $\lambda_2 = 500$ nm, the kinetic energy of the photoelectron is $K_2 = 0.972 \times 10^{-19}$ J or 0.6075 eV.
- It is known that the PE effect formula is given by:

$$K_{\max} = h \frac{c}{\lambda} - W_0 \quad (1)$$

[i]

- Consequently,

$$K_1 = h \frac{c}{\lambda_1} - W_0 \quad (2)$$

$$K_2 = h \frac{c}{\lambda_2} - W_0 \quad (3)$$

- Solve the Eqs. (2) and (3) to obtain the expression for h .

$$\begin{aligned} (2) - (3): \quad & (K_1 - K_2) = hc \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \\ h = \frac{1}{c} & (3 \times 10^{-19} \text{ J} - 0.972 \times 10^{-19} \text{ J}) \left(\frac{(331 \times 10^{-9}) \times (500 \times 10^{-9})}{(500 \times 10^{-9}) - (331 \times 10^{-9})} \right) \\ & = \frac{1}{c} (2.028 \times 10^{-19} \text{ J}) (9.793 \times 10^{-7} \text{ m}) \\ & = 6.62 \times 10^{-34} \text{ J s} \end{aligned} \quad (4)$$

[ii]

- The PE effect formula can be written as:

$$\begin{aligned} K_{\max} &= h \frac{c}{\lambda} - W_0 = h \frac{c}{\lambda} - h \frac{c}{\lambda_0} \\ &= hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \end{aligned} \quad (5)$$

Where $\lambda_0 =$ threshold wavelength.

- The threshold wavelength can be obtained by solving Eq. (2) or Eq. (3).
- Let use the unit of eV and solve the Eq. (2).

$$1.875 \text{ eV} = 1240 \text{ eV} \cdot \text{nm} \left(\frac{1}{331} - \frac{1}{\lambda_0} \right)$$

$$\frac{1.875 \text{ eV}}{1240 \text{ eV} \cdot \text{nm}} = \left(\frac{1}{331} - \frac{1}{\lambda_0} \right)$$

$$1.512 \times 10^{-3} \text{ nm}^{-1} = \left(\frac{1}{331} - \frac{1}{\lambda_0} \right)$$

$$\frac{1}{\lambda_0} = (3.021 \times 10^{-3} - 1.512 \times 10^{-3}) \text{ nm}^{-1}$$

$$\frac{1}{\lambda_0} = 1.509 \times 10^{-3} \text{ nm}^{-1}$$

$$\lambda_0 = 662.691 \text{ nm}$$

[iii]

- The work function for the photocathode is:

$$\begin{aligned} W_0 &= h \frac{c}{\lambda_0} \\ &= \frac{1240 \text{ eV} \cdot \text{nm}}{662.691 \text{ nm}} \\ &= 1.871 \text{ eV} \end{aligned}$$

[3] [b]

- Answer:
- Here, $E = 0.700 \text{ MeV}$, $\lambda = hc/E = 1.771 \times 10^{-12} \text{ m}$.
- The Compton shift formula is:

$$\begin{aligned}\lambda' - \lambda &= \frac{h}{m_0 c} (1 - \cos \theta) \\ &= 2.426 \times 10^{-12} (1 - \cos 2\phi) \\ \lambda' &= \lambda + 2.426 \times 10^{-12} (1 - \cos 2\phi) \quad (1) \\ \lambda' &= 1.771 \times 10^{-12} + 2.426 \times 10^{-12} - 2.426 \times 10^{-12} \cos 2\phi \\ \lambda' &= 4.197 \times 10^{-12} - 2.426 \times 10^{-12} \cos 2\phi\end{aligned}$$

- Divide Eq. (1) by λ :

$$\frac{\lambda'}{\lambda} = \frac{4.197 \times 10^{-12} - 2.426 \times 10^{-12} \cos 2\phi}{1.771 \times 10^{-12}} \quad (2)$$

$$= 2.370 - 1.370 \cos 2\phi$$

- Use the principle of Conservation of Linear Momentum.
Initial P = Final P

- Resolve the momentum into component-x and -y
- Component -x:

$$\begin{aligned}P + 0 &= P' \cos 2\phi + P'_e \cos \phi \\ P'_e \cos \phi &= P - P' \cos 2\phi \quad (3)\end{aligned}$$

- (ii) Component -y:

$$\begin{aligned}0 + 0 &= P' \sin 2\phi - P'_e \sin \phi \\ P'_e \sin \phi &= P' \sin 2\phi \quad (4)\end{aligned}$$

- Solve the Eqs. (3) and (4) for angle ϕ .

$$\frac{(4)}{(3)} \quad \tan \phi = \frac{P' \sin 2\phi}{P - P' \cos 2\phi} \quad (5)$$

$$= \frac{\sin 2\phi}{P/P' - \cos 2\phi}$$

- Since $P = hc/\lambda$, and $P' = hc/\lambda'$

$$\tan \phi = \frac{\sin 2\phi}{(\lambda'/\lambda) - \cos 2\phi} \quad (6)$$

- Rearrange Eq. (6) and simplify it:

$$\begin{aligned}(\lambda'/\lambda) - \cos 2\phi &= \frac{\sin 2\phi}{\tan \phi} \\ (\lambda'/\lambda) - (2\cos^2 \phi - 1) &= \frac{\cos \phi}{\sin \phi} (2\cos \phi \sin \phi) \quad (7) \\ \lambda'/\lambda + 1 &= 4\cos^2 \phi\end{aligned}$$

- Replace Eq. (2) into Eq. (7):

$$\begin{aligned}\lambda'/\lambda + 1 &= 4\cos^2 \phi \\ (2.370 - 1.370 \cos 2\phi) + 1 &= 4\cos^2 \phi \\ 2.370 - 1.370(2\cos^2 \phi - 1) + 1 &= 4\cos^2 \phi \\ 2.370 - 2.740 \cos^2 \phi + 1.370 + 1 &= 4\cos^2 \phi \\ 4.740 &= 4\cos^2 \phi + 2.740 \cos^2 \phi \quad (8) \\ 4.740 &= 6.740 \cos^2 \phi \\ \cos \phi &= \sqrt{\frac{4.740}{6.740}} \\ &= 0.7033 \\ \phi &= 45.31^\circ\end{aligned}$$

[i]

- The scattering angle for the electron is: $\phi = 45.31^\circ$.

[ii]

- The kinetic energy of the recoiling electron is given by the Conservation of energy, i.e.:

$$K'_e = E - E' \quad (9)$$

Where E and E' are the energy of the incident and scattered photon.

- Based on the calculation in [i], the energy of the scattered photon is 0.2936 MeV.

- Consequently,

$$K'_e = 0.7 \text{ MeV} - 0.2936 \text{ MeV} = 0.4064 \text{ MeV} \quad (10)$$

- By substituting the relativistic formula for the kinetic energy into Eq.(10) and solve for the speed of the electron:

$$\begin{aligned}
 K'_e &= E - E_0 = mc^2 - m_0c^2 \\
 (\gamma - 1)m_0c^2 &= 0.4064 \text{ MeV} \\
 (\gamma - 1) &= \frac{0.4064 \text{ MeV}}{0.5110 \text{ MeV}} \\
 &= 0.7953 \tag{9} \\
 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} &= 1.7953 \\
 v &= 0.8305c
 \end{aligned}$$

- Hence, the speed of the recoiled electron is $v = 0.8305c = 2.4915 \times 10^8 \text{ m s}^{-1}$.

[4] [a]

Answer:

- The relativistic electron's de Broglie wavelength is given by:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\gamma m_e v} \tag{1}$$

- The Compton wavelength is:

$$\lambda_c = h/(m_e c) \tag{2}$$

- The total energy of the electron is given by

$$E^2 = p^2 c^2 + m_e^2 c^4 \tag{3}$$

- By rearranging Eq. (3)

$$\begin{aligned}
 p^2 c^2 &= E^2 - m_e^2 c^4 \\
 pc &= \sqrt{E^2 - m_e^2 c^4} \tag{4}
 \end{aligned}$$

- Multiple the right hand side of Eqs. (1) and (2) with c/c :

$$\therefore \lambda = \frac{hc}{pc} = \frac{hc}{\sqrt{E^2 - m_e^2 c^4}} \tag{5}$$

$$\therefore \lambda_c = \frac{hc}{m_e c^2} \tag{6}$$

- Consequently, the ratio of the Compton wavelength to the de Broglie wavelength for a relativistic electron is:

(6) ÷ (5)

$$\begin{aligned}
 \frac{\lambda_c}{\lambda} &= \frac{hc}{m_e c^2} \times \frac{\sqrt{E^2 - m_e^2 c^4}}{hc} \\
 &= \frac{\sqrt{E^2 - m_e^2 c^4}}{m_e c^2} = \sqrt{\frac{E^2 - m_e^2 c^4}{m_e^2 c^4}} \tag{Proven} \\
 &= \sqrt{\left(\frac{E}{m_e c^2}\right)^2 - 1}
 \end{aligned}$$

[4] [a]

Answer:

- Given that $\lambda = 1.00 \times 10^{-11} \text{ m} = 0.01 \text{ nm}$. Needs to find E_k .
- Electron's de Broglie wavelength is given by:

$$\lambda = h/(2m_e E_k)^{\frac{1}{2}} \tag{1}$$

[i]

- The minimum kinetic energy required for the electrons is:

$$\begin{aligned}
 E_k &= \frac{h^2}{2m_e \lambda^2} = \frac{h^2}{2m_e \lambda^2} \times \frac{c^2}{c^2} = \frac{h^2 c^2}{2m_e c^2 \lambda^2} \\
 &= \frac{(1240 \text{ eV} \cdot \text{nm})^2}{2(0.511 \text{ MeV})(0.01 \text{ nm})^2} \\
 &= 15.045 \text{ keV}
 \end{aligned}$$

[ii]

- The minimum photon energy needed to obtain the required resolution is:

$$\begin{aligned}
 E_k &= \frac{hc}{\lambda} \\
 &= \frac{1240 \text{ eV} \cdot \text{nm}}{0.01 \text{ nm}} \\
 &= 124.00 \text{ keV}
 \end{aligned}$$

[5] [a]

Answer:

- The energy of the photon emitted when the electron goes from $n_i \rightarrow n_f$ is:

$$E = \frac{hc}{\lambda} = E_i - E_f = E_1 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

Where $E_1 = \text{Ground state energy} = -13.6 \text{ eV}$

- Given that the photon emitted is in the visible region. Then, it must be in the Balmer series ($n_i \rightarrow n_2$).
- Therefore, the final state is $n = 2$. Now, we need to solve the initial state, n_i :

$$E = \frac{1240 \text{ eV} \cdot \text{nm}}{410.1 \text{ nm}} = -13.6 \text{ eV} \left(\frac{1}{n_i^2} - \frac{1}{2^2} \right)$$

$$3.0237 \text{ eV} = -13.6 \text{ eV} \left(\frac{1}{n_i^2} - \frac{1}{4} \right)$$

$$-0.2223 = \frac{1}{n_i^2} - 0.25$$

$$n_i = \sqrt{\frac{1}{.00277}} = 6.0084$$

- Consequently, the 410.1 nm photon is emitted when the electron goes from $n = 6$ to $n = 2$.
- Or by using the following formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \quad n_f = 2; \quad n_i = 3, 4, 5, \dots$$

- Hence, the final state is, $n_f = 2$.
- Solving the above equation for n_i :

$$\frac{1}{410.1 \times 10^{-9}} = 1.097 \times 10^7 \text{ m}^{-1} \left(\frac{1}{2^2} - \frac{1}{n_i^2} \right)$$

$$0.2223 = \frac{1}{4} - \frac{1}{n_i^2}$$

$$\frac{1}{n_i^2} = 0.25 - 0.2223 = 0.0277$$

$$n_i = 6.008$$

- Since n_i must be an integer, $n_i = 6$.

[5] [a]

Answer:

- To excite an electron in a hydrogen atom from the ground state, energy need is given by:

$$\Delta E = E_f - E_i = -13.6 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \text{ eV} = -13.6 \left(\frac{1}{n_f^2} - \frac{1}{1} \right) \text{ eV}$$

- So, need to find out the highest state that the electron can be excited by the electron of energy 13.0 eV.

$n_1 \rightarrow n_2$

$$\Delta E = E_2 - E_1 = -13.6 \left(\frac{1}{2^2} - \frac{1}{1^2} \right) \text{ eV} = 10.2 \text{ eV}$$

$n_1 \rightarrow n_3$

$$\Delta E = E_3 - E_1 = -13.6 \left(\frac{1}{3^2} - \frac{1}{1^2} \right) \text{ eV} = 12.09 \text{ eV}$$

$n_1 \rightarrow n_4$

$$\Delta E = E_4 - E_1 = -13.6 \left(\frac{1}{4^2} - \frac{1}{1^2} \right) \text{ eV} = 12.75 \text{ eV}$$

$n_1 \rightarrow n_5$

$$\Delta E = E_5 - E_1 = -13.6 \left(\frac{1}{5^2} - \frac{1}{1^2} \right) \text{ eV} = 13.06 \text{ eV}$$

- Consequently, the maximum amount of energy the incident electron can lose inelastically in this collision is 12.75 eV.

$$E_{\text{max,lose}} = 12.75 \text{ eV}$$

[ii]

- The remaining energy of the electron is $= 13 \text{ eV} - 12.75 \text{ eV} = 0.25 \text{ eV}$.

[iii]

- The spectral lines that the hydrogen atoms emit under these conditions can be obtained by using:

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{E \text{ eV}}$$

Hence, the spectral lines that the hydrogen atoms emitted are

$n_1 \rightarrow n_2$

$$\lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{10.2 \text{ eV}} = 121.569 \text{ nm}$$

$n_1 \rightarrow n_3$

$$\lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{12.09 \text{ eV}} = 102.564 \text{ nm}$$

$$n_1 \rightarrow n_4$$

$$\lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{12.75 \text{ eV}} = 97.255 \text{ nm}$$

Q6. J.R. Taylor et al, page 151.

[a]
Mechanical stability: Coulombic attraction supplies the centripetal force,

$$\frac{ke^2}{r^2} = \frac{mv^2}{r}$$

$$\text{Hence, } K = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$$

$$\text{Potential energy, } U = -\frac{ke^2}{r} = -2\frac{ke^2}{2r} = -2K$$

[b]
 $L = n\hbar = mvr$.

$$\text{Also, } \frac{ke^2}{r} = mv^2$$

$$\rightarrow v \cdot mrv = v \cdot n\hbar = ke^2$$

$$\rightarrow v = \frac{ke^2}{n\hbar} \rightarrow r = \frac{n\hbar}{mv} = \frac{n\hbar}{m\left(\frac{ke^2}{n\hbar}\right)} = \frac{n^2\hbar^2}{mke^2} = \frac{4\pi\epsilon_0 n^2 \hbar^2}{me^2} \equiv r_n$$

$$[c] a_0 = r_n(n=1) = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = 0.0529 \text{ nm}$$

Q7. **Solution**

[a]

Conservation of energy-momentum: $E^2 = m^2c^4 + p^2c^2$.

Total relativistic energy is the sum of kinetic energy and rest energy, $E = K + mc^2$.

Combining both, $E = \sqrt{m^2c^4 + p^2c^2} = mc^2 + K$. Squaring, we obtain the relation between kinetic energy and momentum, $pc = \sqrt{2Kmc^2 + K}$.

Now, both electron and proton have the same kinetic energy K , but their momentums are different, so we have, for each of them respectively,

$$p_e c = \sqrt{2Km_e c^2 + K}, \quad p_p c = \sqrt{2Km_p c^2 + K}$$

The ratio of their respective de Broglie wavelengths are then given by

$$\frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} = \frac{\sqrt{2Km_p c^2 + K}}{\sqrt{2Km_e c^2 + K}} = \frac{\sqrt{2m_p c^2 + K}}{\sqrt{2m_e c^2 + K}}$$

[b]

In the non-relativistic limit, $K \ll m_p c^2, m_e c^2$, and

$$\frac{\lambda_e}{\lambda_p} = \frac{\sqrt{2m_p c^2 + K}}{\sqrt{2m_e c^2 + K}} \text{ reduces to } \rightarrow \sqrt{\frac{2m_p c^2}{2m_e c^2}} = \sqrt{\frac{m_p}{m_e}}$$

Q8. **Solution**

(a) For particle in an infinite box, $\frac{n\lambda}{2} = L$, so $p = \frac{h}{\lambda} = \frac{n\hbar}{2L}$.

Energy levels are given by the relativistic total energy,

$$E_n = \left[\left(\frac{n\hbar c}{2L} \right)^2 + (mc^2)^2 \right]^{1/2}$$

The relativistic kinetic energy is given by

$$K_n = E_n - mc^2 = \left[\left(\frac{n\hbar c}{2L} \right)^2 + (mc^2)^2 \right]^{1/2} - mc^2$$

Note: In part (c), the comparison is made between the non-relativistic kinetic energy and the relativistic kinetic energy. Hence, the 'energy levels' in part (a) actually is referring to the relativistic kinetic energy rather than relativistic total energy. However, since the question in (a) does not define clearly what it meant by the 'energy levels', (i.e. whether they refer to the relativistic kinetic energy or the total relativistic energy), candidate who gives either the relativistic energy or the relativistic kinetic energy, or both, will be accepted as correct answers.

(b) Taking $n = 1$ we find

$$E_1 = \left[\left(\frac{hc}{2L} \right)^2 + (mc^2)^2 \right]^{1/2} = \left[\left(\frac{hc}{2 \left(\frac{h}{2mc} \right)} \right)^2 + (mc^2)^2 \right]^{1/2} = [m^2c^4 + m^2c^4]^{1/2} = \sqrt{2}mc^2$$

$$K_1 = E_1 - mc^2 = (\sqrt{2} - 1)mc^2.$$

Note: Due to the ambiguity in part (a), candidate who gives either E_1 or K_1 or both will be accepted as correct answer.

(c) The nonrelativistic (a.k.a. classical) energy in ground state is purely comprised of kinetic energy (note that there is no rest energy in classical physics), given by $K_{1, NR} = \frac{h^2}{8mL^2}$.

Comparing this with K_1 , we see that

$$\frac{K_1 - K_{1, NR}}{K_1} = \frac{(\sqrt{2} - 1)mc^2 - \frac{mc^2}{2}}{(\sqrt{2} - 1)mc^2} = \frac{\sqrt{2} - \frac{3}{2}}{\sqrt{2} - 1} = -0.207$$

The error of the nonrelativistic kinetic energy value is 20.7 % too large compared to the relativistic (correct) one.

Note: Due to the ambiguity in part (a), candidate who gives the answer in terms of

$$\frac{E_1 - K_{1, NR}}{E_1} = \frac{\sqrt{2}mc^2 - \frac{mc^2}{2}}{\sqrt{2}mc^2} = \frac{\sqrt{2} - \frac{1}{2}}{\sqrt{2}} = 0.647 \text{ will also be accepted as correct answer.}$$