Normalised ψ_n (ξ) for simple harmonic oscillator.

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In[28]:= MEnO[n_, a0_, a1_, lastj_] := Module[{},
   K = 2 n + 1;
   a[1] = a1; a[0] = a0;

   For[j = 0, j ≤ lastj, j++,
    a[j + 2] = ((2 j + 1) - (K)) / ((j + 1) (j + 2)) a[j];
   ]; (*end for *)

   If[OddQ[n],
    For[j = 0, j ≤ lastj, j += 2,
      a[j] = 0;
     ];(*end for*)
   ]; (*end if j*)

   If[EvenQ[n],
    For[j = 1, j ≤ lastj, j += 2,
      a[j] = 0;
     ];(*end for *)
   ];(*end if *)
  ];

h[xi_, n_] := Module[{nlast = n},
   a0 = 1; a1 = 1;
   jlast = nlast;
   MEnO[nlast, a0, a1, jlast];
   Sum[a[i] xi^i, {i, 0, nlast}]
  ];

psi[xi_, n_] := h[xi, n] * Exp[-(xi^2) / 2.0];
(*Module[{nlast=n},
  a0=1;a1=1;
  jlast=nlast;
  MEnO[nlast,a0,a1,jlast];
  Exp[-(xi^2)/2.0]*Sum[a[i]xi^i,{i,0,nlast}]
 ];*)

nlast = 10;

For[n = 0, n ≤ nlast, n++,
  A[n] = 1.0 / Sqrt[NIntegrate[psi[xi, n]^2, {xi, -Infinity, Infinity}]];
 ];

Npsi[xi_, n_] := A[n] * psi[xi, n];

Manipulate[Plot[h[xi, n], {xi, -10, 10}, PlotRange → {-20, 20}, AxesLabel → {"ξ", "hn(ξ)"},
  PlotLabel → {"n=" <> ToString[n] <> "; hn(ξ)=", h[ξ, n]}
 ], {n, 0, nlast, 1}]

Manipulate[Plot[(Npsi[xi, n])^2, {xi, -10, 10}, PlotRange → {-1, 1}, AxesLabel → {"ξ", "|ψn(ξ)|^2"},
  PlotLabel → {"n=" <> ToString[n] <> "; ψn(ξ)=", psi[ξ, n]}
```

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], {n, 0, nlast, 1}]

Print["Limit[h[xi,0],{xi→Infinity}]=", Limit[h[xi, 10], {xi → Infinity}]];
```
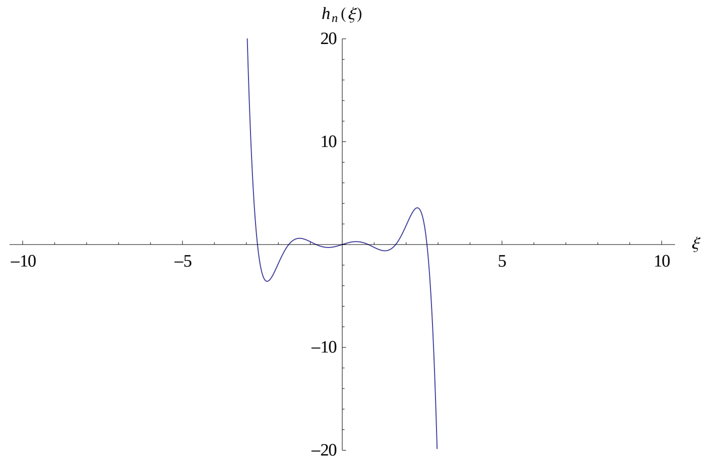

Out[34]=

n

7

$$\left\{n=7;h_n(\xi)=,\ -\frac{8\,\xi^7}{105}+\frac{4\,\xi^5}{5}-2\,\xi^3+\xi\right\}$$

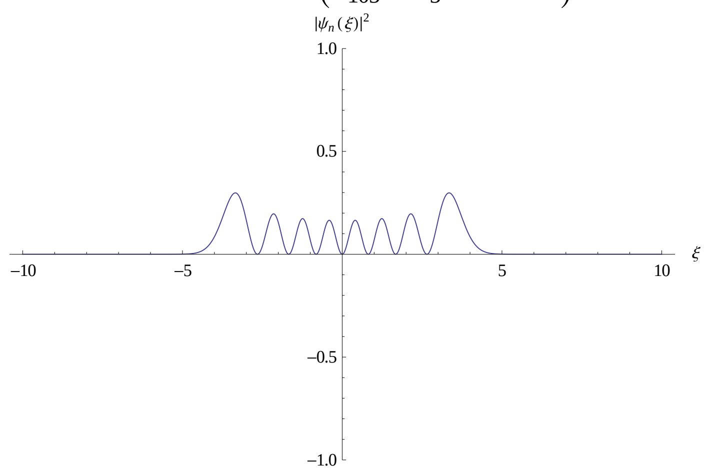


Out[35]=

n

7

$$\left\{n=7;\psi_n(\xi)=,\ e^{-0.5\,\xi^2}\left(-\frac{8\,\xi^7}{105}+\frac{4\,\xi^5}{5}-2\,\xi^3+\xi\right)\right\}$$



```
Limit[h[xi,0],{xi→Infinity}]={-∞}
```

In[37]:=
```
For[n = 0, n ≤ nlast, n++,
  Print["n=", n];
  Print["Aₙ=", A[n]];
  Print["hₙ(ξ)=", h[ξ, n]];
  Print["ψₙ(ξ)=", Npsi[ξ, n]];
  Print[" "];
 ];
```

n=0

$A_n = 0.751126$

$h_n(\xi) = 1$

$\psi_n(\xi) = 0.751126\, e^{-0.5\,\xi^2}$

n=1

$A_n = 1.06225$

$h_n(\xi) = \xi$

$\psi_n(\xi) = 1.06225\, e^{-0.5\,\xi^2}\,\xi$

n=2

$A_n = 0.531126$

$h_n(\xi) = 1 - 2\,\xi^2$

$\psi_n(\xi) = 0.531126\, e^{-0.5\,\xi^2}\left(1 - 2\,\xi^2\right)$

n=3

$A_n = 1.30099$

$h_n(\xi) = \xi - \frac{2\,\xi^3}{3}$

$\psi_n(\xi) = 1.30099\, e^{-0.5\,\xi^2}\left(\xi - \frac{2\,\xi^3}{3}\right)$

n=4

$A_n = 0.459969$

$h_n(\xi) = 1 - 4\,\xi^2 + \frac{4\,\xi^4}{3}$

$\psi_n(\xi) = 0.459969\, e^{-0.5\,\xi^2}\left(1 - 4\,\xi^2 + \frac{4\,\xi^4}{3}\right)$

n=5

$A_n = 1.45455$

$h_n(\xi) = \xi - \frac{4\xi^3}{3} + \frac{4\xi^5}{15}$

$\psi_n(\xi) = 1.45455\, e^{-0.5\,\xi^2}\left(\xi - \frac{4\xi^3}{3} + \frac{4\xi^5}{15}\right)$

n=6

$A_n = 0.419892$

$h_n(\xi) = 1 - 6\xi^2 + 4\xi^4 - \frac{8\xi^6}{15}$

$\psi_n(\xi) = 0.419892\, e^{-0.5\,\xi^2}\left(1 - 6\xi^2 + 4\xi^4 - \frac{8\xi^6}{15}\right)$

n=7

$A_n = 1.57109$

$h_n(\xi) = \xi - 2\xi^3 + \frac{4\xi^5}{5} - \frac{8\xi^7}{105}$

$\psi_n(\xi) = 1.57109\, e^{-0.5\,\xi^2}\left(\xi - 2\xi^3 + \frac{4\xi^5}{5} - \frac{8\xi^7}{105}\right)$

n=8

$A_n = 0.392773$

$h_n(\xi) = 1 - 8\xi^2 + 8\xi^4 - \frac{32\xi^6}{15} + \frac{16\xi^8}{105}$

$\psi_n(\xi) = 0.392773\, e^{-0.5\,\xi^2}\left(1 - 8\xi^2 + 8\xi^4 - \frac{32\xi^6}{15} + \frac{16\xi^8}{105}\right)$

n=9

$A_n = 1.66639$

$h_n(\xi) = \xi - \frac{8\xi^3}{3} + \frac{8\xi^5}{5} - \frac{32\xi^7}{105} + \frac{16\xi^9}{945}$

$\psi_n(\xi) = 1.66639\, e^{-0.5\,\xi^2}\left(\xi - \frac{8\xi^3}{3} + \frac{8\xi^5}{5} - \frac{32\xi^7}{105} + \frac{16\xi^9}{945}\right)$

n=10

$A_n = 0.372617$

$h_n(\xi) = 1 - 10\xi^2 + \frac{40\xi^4}{3} - \frac{16\xi^6}{3} + \frac{16\xi^8}{21} - \frac{32\xi^{10}}{945}$

$$\psi_n(\xi) = 0.372617\, e^{-0.5\, \xi^2} \left(1 - 10\,\xi^2 + \frac{40\,\xi^4}{3} - \frac{16\,\xi^6}{3} + \frac{16\,\xi^8}{21} - \frac{32\,\xi^{10}}{945}\right)$$