A monocromatic 1D wave is described by the formula

$y[x, t] = \text{Sin}\left[k\ x + \omega t\right]$, $\omega = \frac{2\pi}{\lambda}$ is the wave number; ω = angular frequency.

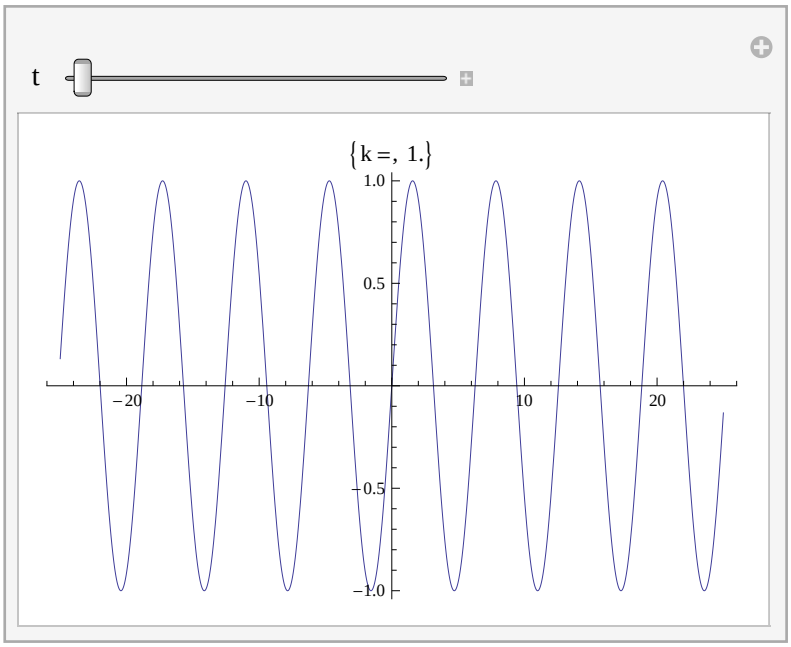
Let $k_i = k_0 + i\ \Delta k$, $\Delta k << 1$, say 0.001. We assume k_0=1.

In[2]:=
```
(* Nk Number of wave to add *)
xmax = 200;
Deltak = 0.001;
k[0] = 1.0; omega = 0.1;
k[i_] := k[0] + i * Deltak;

y[i_, x_, t_] := Sin[k[i] * x - omega * t];(* y is a monochromatic wave with wavenumber k[i] *)
Y[Nk_, x_, t_] := Sum[y[i, x, t], {i, 0, Nk}];
(*Y is the wavepacket formed by superiposing Nk waves *)
```

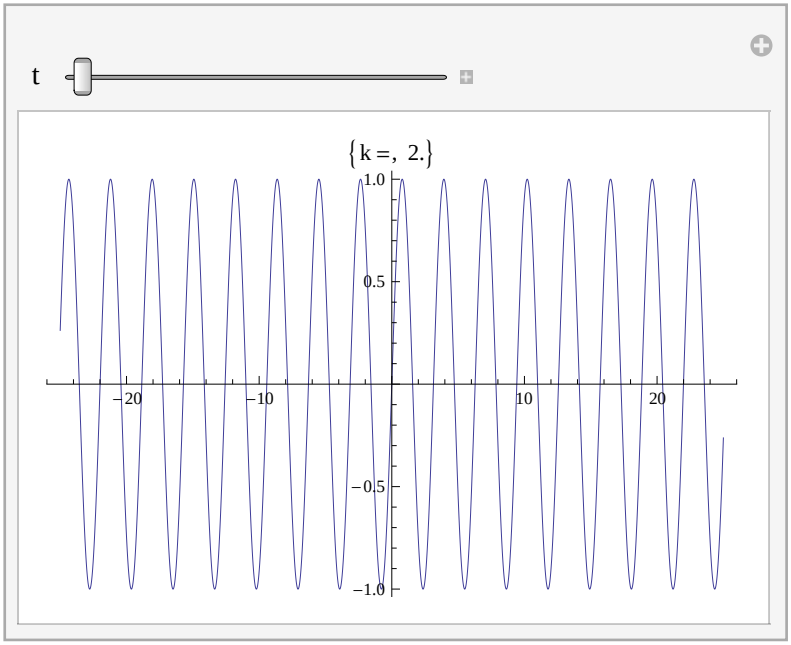
Here we ask *Mathematica* to show the wave form $y[x, t] = \text{Sin}\left[k\ x + \omega t\right]$ for $k = k_0$ and $k = k_0 + N_k\ \Delta k = 2.0$, N_k =1000.

In[8]:=
```
j = 0;
Manipulate[
 Plot[y[j, x, t], {x, -xmax/8, xmax/8}, PlotLabel → {"k=", k[j]}], {t, 0, 100, 0.1}
]

kk = 1000;
Manipulate[
 Plot[y[kk, x, t], {x, -xmax/8, xmax/8}, PlotLabel → {"k=", k[kk]}], {t, 0, 100, 0.1}
]
```

Out[9]=



Out[11]=

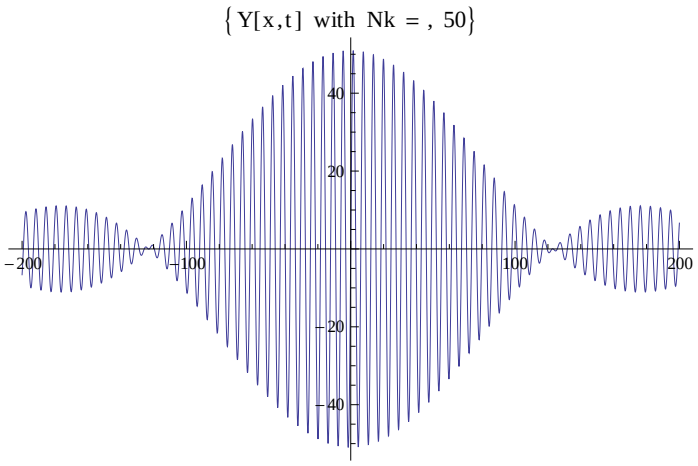


Here we ask *Mathematica* to show the wave form $Y[x,t]=\sum_{i=0}^{i=N_k} \text{Sin}\left[k_i\, x+\omega t\right]$ at t=0 for $N_k = 50, 100, 200, 500$. You will observe that the large the number of wave we add to form Y[x,t], the more

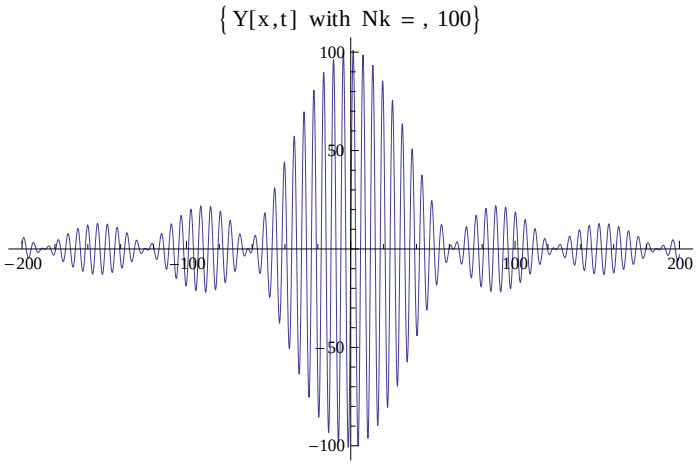
localise it becomes.

In[12]:=
```
Nk = 50;
Plot[Y[Nk, x, 0], {x, -xmax, xmax}, PlotRange → All, PlotLabel → {"Y[x,t] with Nk = ", Nk}]
Nk = 100;
Plot[Y[Nk, x, 0], {x, -xmax, xmax}, PlotRange → All, PlotLabel → {"Y[x,t] with Nk = ", Nk}]
Nk = 200;
Plot[Y[Nk, x, 0], {x, -xmax, xmax}, PlotRange → All, PlotLabel → {"Y[x,t] with Nk = ", Nk}]
Nk = 500;
Plot[Y[Nk, x, 0], {x, -xmax, xmax}, PlotRange → All, PlotLabel → {"Y[x,t] with Nk = ", Nk}]
```

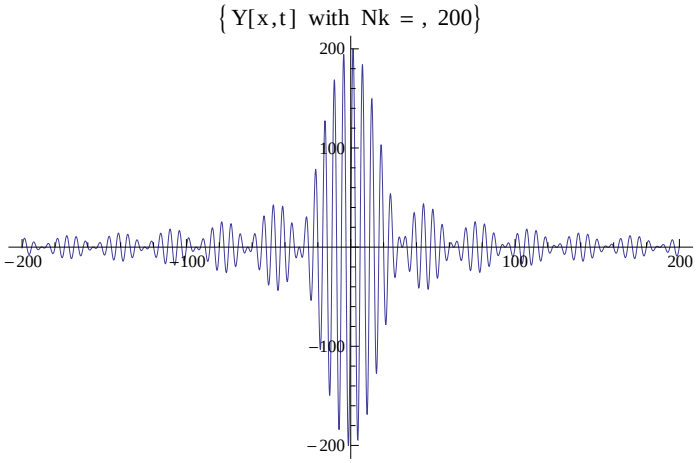
Out[13]=



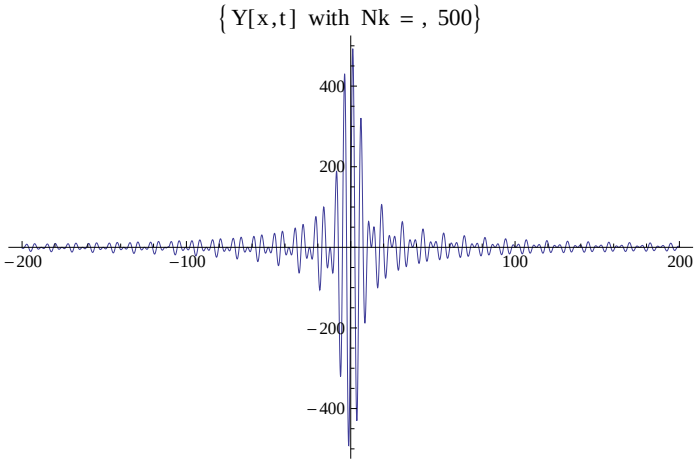
Out[15]=



Out[17]=



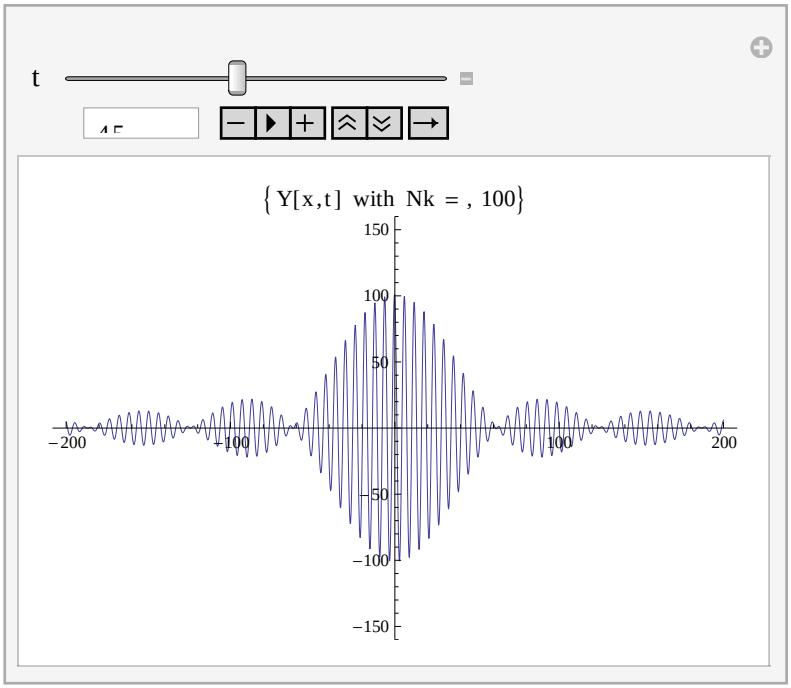
Out[19]=



In the following, we ask *Mathematica* to display a vidoe showing the progressive motion of the wave form $Y[x, t]$ for an extensive period of time, from t=0 to t = 100.

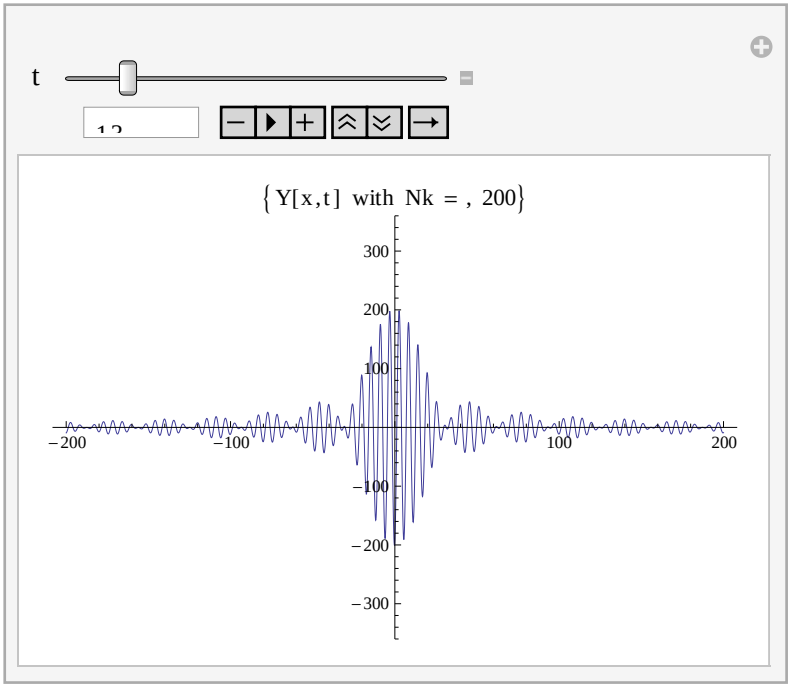
In[61]:=
```
Manipulate[Nk = 100;
 Plot[Y[Nk, x, t], {x, -xmax, xmax},
  PlotRange → {-160, 160}, PlotLabel → {"Y[x,t] with Nk = ", Nk}]
 , {t, 0, 100, 1}]
```

Out[61]=



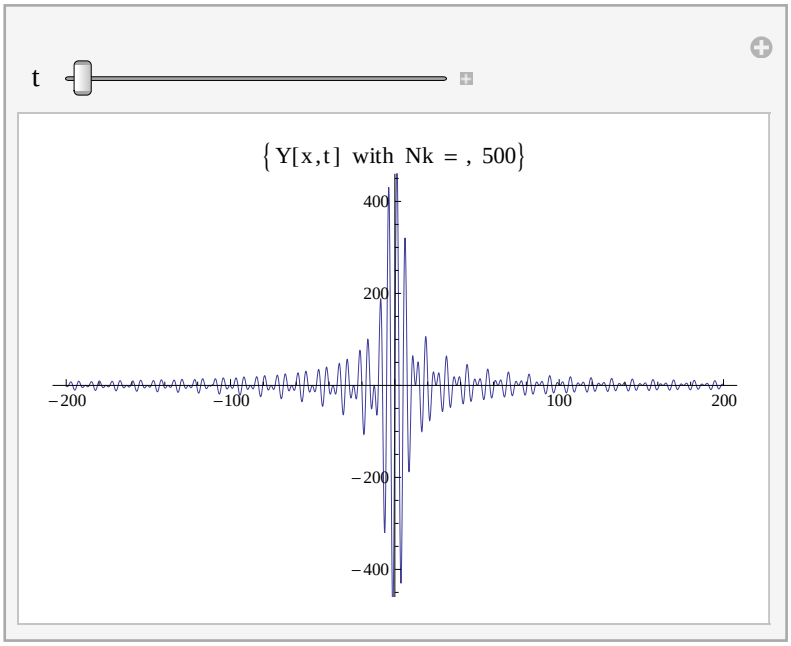
In[63]:=
```
Manipulate[Nk = 200;
 Plot[Y[Nk, x, t], {x, -xmax, xmax},
  PlotRange → {-360, 360}, PlotLabel → {"Y[x,t] with Nk = ", Nk}]
 , {t, 0, 100, 1}]
```

Out[63]=



In[23]:=
```
Manipulate[Nk = 500;
 Plot[Y[Nk, x, t], {x, -xmax, xmax},
  PlotRange → {-460, 460}, PlotLabel → {"Y[x,t] with Nk = ", Nk}]
 , {t, 0, 100, 1}]
```

Out[23]=



In[21]:=
```
xmax
```

Out[21]=
```
200
```