Solution for Q1 (cont.2)

Now, solve for C_n as in

$$\psi(x,t) = \sum_n C_n \psi_n(x) e^{-iE_n t/\hbar}.$$

Set $t=0$,

$$\psi(x,0) = \sum_n C_n \psi_n(x) = A(x-a/2)(x+a/2)$$
$$= A\left(x^2 - \frac{a^2}{4}\right)$$

Normalization $\displaystyle\int_{-a/2}^{a/2} |\psi(x,0)|^2 dx = 1$

$$\therefore \quad A^2 \int_{-a/2}^{a/2} \left(x^2 - \frac{a^2}{4}\right)^2 dx = 1$$

Solution for Q1 (cont.3)

$$A^2 \int_{-a/2}^{a/2} \left(x^2 - \frac{a^2}{4}\right)^2 dx = 1$$

$$A^2 \int_{-a/2}^{a/2} \left(x^4 - \frac{a^2x^2}{2} + \frac{a^4}{16}\right) dx = A^2\left[\frac{x^5}{5} - \frac{a^2x^3}{6} + \frac{a^4}{16}x\right]_{-a/2}^{+a/2}$$

$$= \cancel{A^2} \left[\cancel{-\frac{2a^5}{5.32} + 2\frac{a^2 a^3}{6.8} + \frac{2.a^4}{16}\left(\frac{a}{2}\right)} \right.$$

$$= -A^2\left[\frac{-a^5}{5.32} + \frac{a^2}{6}\cdot\frac{a^3}{8} + \frac{a^4}{16}\cdot\left(\frac{-a}{2}\right) - \frac{a^5}{5.32} + \frac{a^2a^3}{6.8} - \frac{a^4}{16}\left(\frac{a}{2}\right)\right]$$

$$= -A^2\left[-\frac{a^5}{80} + \frac{a^5}{24} - \frac{a^5}{16}\right] = A^2a^5\left[+\frac{1}{30}\right] = \frac{A^2a^5}{30} = 1$$

$$\Rightarrow \quad A = \sqrt{\frac{30}{a^5}}$$