Tutorial to submit online

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2. Prove that for a wave function that is the solution to the Schroedinger eq
tion, the normalisation of the wave function is time-independent,

$$\frac{d}{dt}\left(\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx\right) = 0.$$

Prove this:

$$\langle p \rangle = \int \Psi^* \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right) \Psi dx.$$

Problem 1.17 A particle is represented (at time $t = 0$) by the wave function

$$\Psi(x, 0) = \begin{cases} A(a^2 - x^2), & \text{if } -a \le x \le +a, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the normalization constant A.

(b) What is the expectation value of x (at time $t = 0$)?

(c) What is the expectation value of p (at time $t = 0$)? (Note that you *cannot* get it from $p = md\langle x\rangle/dt$. Why not?)

(d) Find the expectation value of x^2.

(e) Find the expectation value of p^2.

(f) Find the uncertainty in x (σ_x).

Problem 1.16 Show that

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 \, dx = 0$$

for any two (normalizable) solutions to the Schrödinger equation, Ψ_1 and Ψ_2.