Tutorial 2.1

Q1

Separation of variables

$$i\hbar\frac{1}{\varphi}\frac{d\varphi}{dt} = -\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V(x) = E$$

LHS is a function of t alone while the RHS is a function of x alone. Equation 2.4 is true only if both sides equal to a *constant*.We will call this constant E

$$\frac{d\varphi}{dt} = -\frac{iE}{\hbar}\varphi \qquad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

The solution to the time-dependent part

$$\varphi(t) = e^{-iEt/\hbar}.$$

Exercise: Show this.

Q2

Show that the total solution is a solution to the TDSE

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-itE_n/\hbar}$$

A solution to

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial^2 x} + V\Psi$$

Q3

For an infinite square well,

$$\frac{d^2\psi}{dx^2} = -k^2\psi, \text{ where } k \equiv \frac{\sqrt{2mE}}{\hbar}; k^2 \geq 0$$

E, must be positive

WHY ?

Using Euler relation, show that

$$\psi(x) = C_1 e^{ikx} + C_2 e^{-ikx}$$

$$= A \sin kx + B \cos kx$$

Q4

The TISE solutions are mutually orthogonal

Given ψ_n solution to a tiame-independent Schroedinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi=E\psi$$

$$Exercise:\ Proof\ \int \psi_m(x)^*\psi_n(x)dx=\delta_{mn}$$

Q5

Prove that

$$c_n = \int \psi_n(x)^* f(x) dx$$

This can be simply proven by making use of the orthogonality of the TISE solutions

$$\int \psi_m(x)^* \psi_n(x) dx = \delta_{mn}$$

Q6

Normalisation of c_n

- Use $\int |\Psi(x,t)|^2 = 1$

  and $\int \psi_n^*(x)\psi_m(x)dx = \delta_{mn}$

  to prove

$$\sum_{n=1}^{\infty} |c_n|^2 = 1$$

Q8

Consider the solutions to a quantum harmonic potential

- Assume *n* is 1, write down $h(\xi)$, hence the stationary wave function, $\psi_1(x)$.
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- Assume *n* is 2, write down $h(\xi)$, hence the stationary wave function, $\psi_2(x)$.

Q9

Consider the solutions to a quantum harmonic potential

- Derive H_1, H_2, H_3 from the Rodrigues formula.
- Derive H_3, H_4 from H_1, H_2 using the recursion relation.
- As a check, the function H_3 derived using both methods must agree.

Q10

The time-dependent “stationary” solution is a traveling plane wave

$$\Psi_k(x,t) \;=\; Ae^{ik(x-\frac{\hbar k}{2m}t)}$$

SHOW

$$\int_{-\infty}^{\infty} \Psi_k^* \Psi_k dx \rightarrow \infty$$