TEST I (make-up)
ZCT 207 Statistical Mechanics

Name:

Matrix Number:

This test is divided into Part (A) and Part (B), printed on two opposite pages. Write and submit your answers on the question paper. Time: 50 minutes.

2.

(a) (i) How would you determine if the expression

$$\delta a = M(x, y)\,dx + N(x, y)\,dy$$

is an exact differential? $N(x,y)$, $M(x,y)$ are continuous functions of variables x,y.

(ii) State the first law of thermodynamics for the case where the total number of particle in the system is constant.

(iii) Show that heat transferred of the system is not an exact differential.

(iv) Define entropy change in terms of heat transferred of a system.

(v) Show that entropy change as defined in (iv) is an exact differential.

[60 marks]

(b) (i) What is the total energy of an ideal gas system E in terms of temperature and total number of gas particles?

(ii) Derive the heat capacity at constant volume of an ideal gas system from the first law as defined in a(ii)

[40 marks]

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Solution

2(a) (i) If both functions fulfill $\frac{\partial M(x,y)}{\partial y}=0;\frac{\partial N(x,y)}{\partial x}=0$ then δa is an exact differential.

[10 marks]

(a) (ii) $\delta Q = dE + \delta W$

Heat transferred into the system equals the change in the total energy of the system plus work done by the system.

[5 marks]

(a) (iii) From first law,

$dQ = PdV + dE = PdV + (Nfk/2)dT$.

Take $P = P(T, V)$ and use EOS $P(T, V) = nRT/V = NkT/V$

$\Rightarrow dQ = Nk\,[\,(f/2)\,dT + (T/V)\,dV]$.

Compare this to $\delta a = A_x(x, y)dx + A_y(x, y)dy$.

Let $A_x(T,V) \equiv f/2T$; $A_y(T,V) \equiv 1/V$; $x \equiv T$; $y \equiv V$, where V,T are chosen as two independent variables.

$\partial A_x/\partial y \equiv \partial(f/2)/\partial V = 0$; $\partial A_y/\partial x \equiv \partial(T/V)/\partial T = 1/V$.

$\Rightarrow \partial A_x/\partial y = 0 \neq \partial A_y/\partial x = 1/V$

$\Rightarrow$ dQ is not an exact differential.

[10 marks]

(a) (iv) Entropy change $dS = dQ/T$

[5 marks]

(v) $dS = dQ/T = N\,k\,[(f/2T)dT + (1/V)\,dV]$

Let $A_x(T,V) \equiv f/2T$; $A_y(T,V) \equiv 1/V$;

$\partial A_x/\partial y \equiv \partial(f/2T)/\partial V = 0$; $\partial A_y/\partial x \equiv \partial(1/V)/\partial T = 0$.

$\Rightarrow \partial A_x/\partial y = \partial A_y/\partial x = 0$

$\Rightarrow dS = dQ/T$ is an exact differential.

[20 marks]

(b) (i) Total energy of an ideal gas system is: $E = 3NkT/2$

[10 marks]

(ii) From the first law, $dQ = PdV + dE = PdV + (Nfk/2)dT$.

$\Rightarrow dQ = Nk\,[\,(f/2)\,dT + (T/V)\,dV]$.

By definition, $C_V = \partial Q/\partial T|_V \equiv Nk\,\partial/\partial T[(f/2)\,dT + (T/V)\,dV]|_V = Nfk/2$.

For ideal gas, $f = 3$. $\Rightarrow C_V = 3Nk/2$.

[30 marks]