TEST I

ZCT 207 Statistical Mechanics

Solution

a(i) Amit, page 19.

$\gamma = 1 + 2/f \Rightarrow f = 2/(\gamma - 1) = 2/(1.66 - 1) = 3.03$ (5 + 5 marks for both correct steps and answer respectively)

a(ii) Classical physics predicts that the DOF of a diatomic gas is $f = 3r = 6$

(5 + 5 marks for both correct steps and answer respectively)

a(iii) $\gamma = 1.4$ implies a DOF of $f = 2/(1.4 - 1) = 2/(0.4) = 5$

(5 + 5 marks for both correct steps and answer respectively)

a(iv) The discrepancy is due to the fact that some of the DOF is not activated at low kT. To activate a given DOF in the molecule the thermal energy kT has to be at least larger than the quantum gap ΔE associated with that DOF in the molecule.

(10 marks for statement displaying understanding to compare the thermal energy kT with the excitation energy ΔE due to quantum gap.)

b(i) λ has to be negative so that $f(\boldsymbol{v}) \to 0 \; when \; v \to \infty$

(20 marks, both textual and mathematical statements must be presented)

(ii) Amit, page 27, 83.

Use normalisation condition on $f(\mathbf{v})$, ie. $\int_{-\infty}^{\infty} f(\boldsymbol{v}) d^3\boldsymbol{v} = 1$ (statement of normalisation 10 mark)

$$\int_{-\infty}^{\infty} f(\boldsymbol{v}) d^3\boldsymbol{v} = C\int_{-\infty}^{\infty} e^{\lambda v^2} d^3\boldsymbol{v} = C\int_{-\infty}^{\infty} e^{\lambda(v_x^2+v_y^2+v_z^2)} d^3\boldsymbol{v} = C\int_{-\infty}^{\infty} e^{\lambda v_x^2} dv_x \int_{-\infty}^{\infty} e^{\lambda v_y^2} dv_y \int_{-\infty}^{\infty} e^{\lambda v_z^2} dv_z = C\left(\sqrt{\frac{-\lambda}{\pi}}\right)^3$$

(algebra, 20 mark)

$$C = \left(\frac{-\lambda}{\pi}\right)^{3/2}$$

(correct answer 10 mark)