UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama

Sidang Akademik 2014/2015

Januari 2015

ZCT 211/2 – Vector Analysis

[*ZCT 211/2 – Analisis Vektor*]

Duration: 2 hours

[*Masa: 2 jam*]

Please check that this examination paper consists of FOUR (4) printed pages before the examination begins.

[*Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT (4) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*]

Answer all questions. [*Jawab semua soalan.*]

Q1.

(a) (*i*) If **A** and **B** are two non-zero vectors, show that **|A + B|** < **|A|** + **|B|**. **A**, **B** are not parallel to each other.

[*Jika* **A** *dan* **B** adalah *dua vektor bukan-sifar, tunjukkan bahawa* **|A + B|** < **|A|** + **|B|**. **A** dan **B** adalah bukan selari.]

(*ii*) Using the results in (*i*), show that **|A + B + C |** < **|A|** + **|B|**+ **|C|**, where **C** is a non-zero vector.

[*Dengan menggunakan jawapan anda di (i), tunjukkan bahawa* **|A + B + C |** < **|A|** + **|B|**+ **|C|**, *yang* **C** *adalah vektor bukan sifar*.]

(50/100)

(b) (*i*) Determine a unit vector perpendicular to the plane of $\mathbf{A} = 2\,\hat{\imath} - 6\,\hat{\jmath} - 3\,\hat{k}$ and $\mathbf{B} = 4\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k}$.

[*Dapatkan suatu vektor unit yang berserenjang dengan satah yang dibentukkan oleh* $\boldsymbol{A} = 2\,\hat{\imath} - 6\,\hat{\jmath} - 3\,\hat{k}$ *dan* $\boldsymbol{B} = 4\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k}$].

(*ii*) Show that [*Tunjukkan bahawa*]

$$(\mathbf{A} \times \mathbf{B}) \bullet (\mathbf{B} \times \mathbf{C}) \times (\mathbf{C} \times \mathbf{A}) = (\mathbf{A} \bullet \mathbf{B} \times \mathbf{C})^2$$

(50/100)

Q2.

A particle moves so that its position vector is given by $\boldsymbol{r} = \cos\omega t\ \hat{\imath} + \sin\omega t\ \hat{\jmath}$, where ω is a constant.

[*Suatu zarah bergerak sedemikian rupa supaya vektor kedudukannya diberikan oleh* $\boldsymbol{r} = \cos\omega t\ \hat{\imath} + \sin\omega t\ \hat{\jmath}$, *di mana* ω *adalah satu pemalar*.]

(a) Show that [*Tunjukkan bahawa*]

r×**v** is a constant vector, where **v** is the velocity of the particle.

[**r**×**v** *adalah suatu vektor malar, di mana* **v** *adalah halaju zarah itu.*]

(20/100)

(b)(i) Prove $\nabla^2 \frac{1}{r} = 0$, where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ denotes the Laplacian operator.

[*Tunjukkan bahawa* $\nabla^2 \frac{1}{r} = 0$, *yang* $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ *menandakan operator Laplacian*.]

(b) (ii) Show that [*Tunjukkan bahawa*],

$$\nabla \cdot (\varphi \mathbf{A}) = (\nabla \varphi) \cdot \mathbf{A} + \varphi(\nabla \cdot \mathbf{A})$$

where φ is a differentiable scalar function, $\mathbf{A}$ is a differentiable vector.

[*di mana* φ *fungsi skalar terbezakan dan* $\boldsymbol{A}$ *vektor yang terbezakan.*]

(40/100)

(c) Prove that if $\int_{P_1}^{P_2} \boldsymbol{F} \cdot d\boldsymbol{r}$ is independent of the path joining any two points P_1 and P_2 in a given region, then $\oint \boldsymbol{F} \cdot d\boldsymbol{r} = 0$ for all closed paths in the region and conversely.
[*Buktikan bahawa jika* $\int_{P_1}^{P_2} \boldsymbol{F} \cdot d\boldsymbol{r}$ *adalah merdeka daripada lintasan yang menyambungkan titik-titik* P_1 *dan* P_2 *dalam sesuatu rantau, maka* $\oint \boldsymbol{F} \cdot d\boldsymbol{r} = 0$ *untuk semua lintasan tertutup dalam rantau itu, dan sebaliknya.*]

(40/100)

Q3. Given a vector field $\boldsymbol{A}(x, y, z) = 6x\,\hat{\imath} - y\,\hat{\jmath} + 8z\,\hat{k}$.

(a) Calculate the flux of $\boldsymbol{A}$ through the shaded flat surface as shown in the Figure 1.
[*Hitungkan fluks bagi* $\boldsymbol{A}$ *yang melalui permukaan rata seperti yang ditunjukkan di dalam Rajah 1.*]

(70/100)

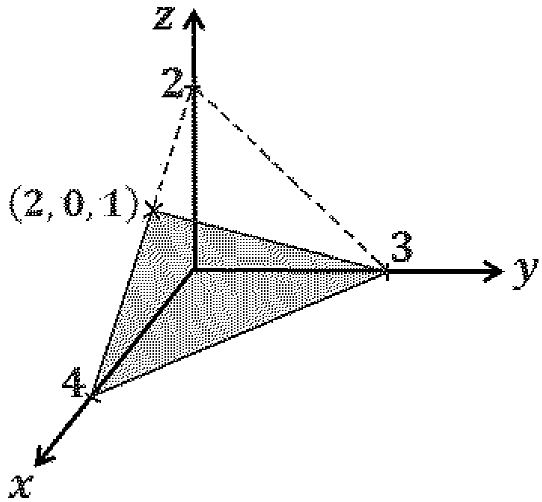

Figure 1
[*Rajah 1*]

(b) Rewrite $\boldsymbol{A}(x, y, z) = 6x\,\hat{\imath} - y\,\hat{\jmath} + 8z\,\hat{k}$ in spherical coordinate system.
[*Tuliskan semula* $\boldsymbol{A}(x, y, z) = 6x\,\hat{\imath} - y\,\hat{\jmath} + 8z\,\hat{k}$ *dalam sistem koordinat sfera.*]

(30/100)

Q4. Given two vector fields $\mathbf{G} = 8yz\,\hat{\imath} - 6xz\,\hat{\jmath} + x^2y\,\hat{k}$ and $\mathbf{H} = 2x^2\,\hat{\imath} + 3y\,\hat{\jmath} + z\,\hat{k}$.
[*Diberi* $\mathbf{G} = 8yz\,\hat{\imath} - 6xz\,\hat{\jmath} + x^2y\,\hat{k}$ *dan* $\boldsymbol{H} = 2x^2\,\hat{\imath} + 3y\,\hat{\jmath} + z\,\hat{k}$.]

(a) With proper labels and arrows, sketched the volume elements in spherical coordinate system.
[Dengan label dan anak-panah yang sesuai, lakarkan elemen isipadu dalam sistem koordinat sfera]

(10/100)

(b) Determine whether **F** and **G** are conservative vector fields.
[*Tentukan sama ada* ***F*** *dan* ***G*** *adalah medan vektor abadi*]

(15/100)

(c) Evaluate the flux of $\nabla \times \mathbf{G}$ through the curved surface of the cylinder as shown in Figure 2.
[*Nilaikan fluks bagi* $\nabla \times \boldsymbol{G}$ *yang melalui permukaan lengkung silinder seperti yang ditunjukkan di dalam Rajah 2*]

(35/100)

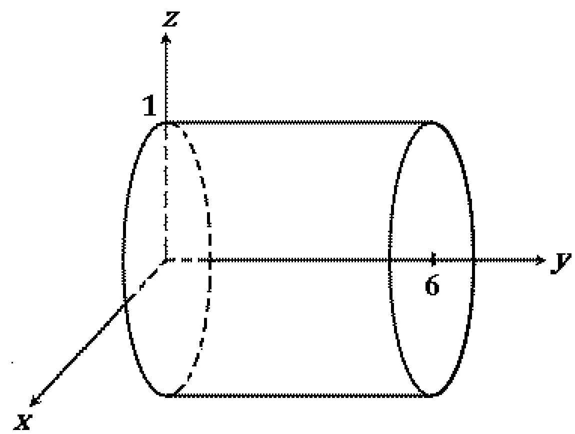

Figure 2. A cylinder with flat surfaces on xz-plane and plane $y = 6$.

[*Rajah 2. Satu silinder dengan permukaan rata pada satah-xz dan satah* $y = 6$]

(d) Use Gauss theorem to calculate the flux of **H** through the curved surface of the hemisphere as shown in Figure 3.
[*Gunakan Teorem Gauss untuk menilaikan fluks bagi* ***H*** *yang melalui permukaan lengkung hemisfera seperti yang ditunjukkan dalam Rajah 3*].

(40/100)

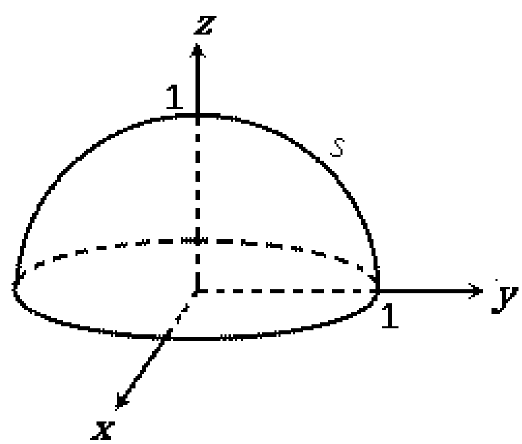

Figure3. Hemisphere : $x^2 + y^2 + z^2 = 1$, $z \geq 0$

[*Rajah 3. Hemisfera* $x^2 + y^2 + z^2 = 1$, $z \geq 0$]

ZCT 211 SE 14/15, sem 1 final exam Solutions:

Note: Marks up to a maximum of 20% (i.e. 20 marks) will be deducted from each question (Q) if vector quantities are not appropriately labeled (e.g., with an arrow sign ("→") on top of the vectorial quantity or a tudor ("~") at the bottom of a vectorial quantity) in your answer scripts.

Q1(a)(*i*) (25 marks)
Q1(a)(*ii*) (25 marks)
Q1(b)(*i*) Q32, page 25, Schaum's series. (25 marks)
Q1(b)(*ii*) Q52, page 30, Schaum's series. (25 marks)

Q2(a) Q12, page 42, Schaum's series. (20 marks)
Q2(b)(i): Q17, page 64, Schaum's series. (20 marks)
Q2(b)(ii): Q18(b), page 65, Schaum's series. (20 marks)
Q2(c): Q13(b), page 93, Schaum's series. (40 marks)