Tutorial 3

Pass up personally
on
Monday class
12 pm, 27 Oct 2014

Tutorial for Chapter 3

32.

Find the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t$, $y = 2\cos 3t$, $z = 8t$ at any time $t > 0$. Find the magnitude of the velocity and acceleration.

Ans. $\mathbf{v} = 6\cos 3t\,\mathbf{i} - 6\sin 3t\,\mathbf{j} + 8\mathbf{k}$, $\mathbf{a} = -18\sin 3t\,\mathbf{i} - 18\cos 3t\,\mathbf{j}$, $|\mathbf{v}| = 10$, $|\mathbf{a}| = 18$

33.

Find the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t$, $y = 2\cos 3t$, $z = 8t$ at any time $t > 0$. Find the magnitude of the velocity and acceleration.

Ans. $\mathbf{v} = 6\cos 3t\,\mathbf{i} - 6\sin 3t\,\mathbf{j} + 8\mathbf{k}$, $\mathbf{a} = -18\sin 3t\,\mathbf{i} - 18\cos 3t\,\mathbf{j}$, $|\mathbf{v}| = 10$, $|\mathbf{a}| = 18$

34. If $\mathbf{A} = t^2\,\mathbf{i} - t\,\mathbf{j} + (2t+1)\mathbf{k}$ and $\mathbf{B} = (2t-3)\mathbf{i} + \mathbf{j} - t\mathbf{k}$, find $(a)\ \frac{d}{dt}(\mathbf{A}\cdot\mathbf{B})$, $(b)\ \frac{d}{dt}(\mathbf{A}\times\mathbf{B})$, $(c)\ \frac{d}{dt}|\mathbf{A}+\mathbf{B}|$, $(d)\ \frac{d}{dt}(\mathbf{A}\times\frac{d\mathbf{B}}{dt})$ at $t=1$.

Ans. (*a*) -6, (*b*) $7\mathbf{j}+3\mathbf{k}$, (*c*) 1,
(*d*) $\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$

38.

If $\frac{d^2\mathbf{A}}{dt^2} = 6t\,\mathbf{i} - 24t^2\,\mathbf{j} + 4\sin t\,\mathbf{k}$, find $\mathbf{A}$ given that $\mathbf{A} = 2\mathbf{i} + \mathbf{j}$ and $\frac{d\mathbf{A}}{dt} = -\mathbf{i} - 3\mathbf{k}$ at $t = 0$.

Ans. $\mathbf{A} = (t^3 - t + 2)\mathbf{i} + (1 - 2t^4)\mathbf{j} + (t - 4\sin t)\mathbf{k}$

44.

If $\mathbf{A} = x^2yz\,\mathbf{i} - 2xz^3\,\mathbf{j} + xz^2\,\mathbf{k}$ and $\mathbf{B} = 2z\,\mathbf{i} + y\,\mathbf{j} - x^2\,\mathbf{k}$, find $\frac{\partial}{\partial x\,\partial y}(\mathbf{A}\times\mathbf{B})$ at $(1,0,-2)$.

Ans. $-4\mathbf{i} - 8\mathbf{j}$

45.

If $\mathbf{C}_1$ and $\mathbf{C}_2$ are constant vectors and λ is a constant scalar, show that $\mathbf{H} = e^{-\lambda x}(\mathbf{C}_1 \sin \lambda y + \mathbf{C}_2 \cos \lambda y)$ satisfies the partial differential equation $\dfrac{\partial^2 \mathbf{H}}{\partial x^2} + \dfrac{\partial^2 \mathbf{H}}{\partial y^2} = \mathbf{0}$.

47.

Find (*a*) the unit tangent $\mathbf{T}$, (*b*) the curvature κ, (*c*) the principal normal $\mathbf{N}$, (*d*) the binormal $\mathbf{B}$, and (*e*) the torsion τ for the space curve $x = t - t^3/3$, $y = t^2$, $z = t + t^3/3$.

Ans. (*a*) $\mathbf{T} = \dfrac{(1-t^2)\mathbf{i} + 2t\,\mathbf{j} + (1+t^2)\mathbf{k}}{\sqrt{2}(1+t^2)}$ (*c*) $\mathbf{N} = -\dfrac{2t}{1+t^2}\mathbf{i} + \dfrac{1-t^2}{1+t^2}\mathbf{j}$ (*e*) $\tau = \dfrac{1}{(1+t^2)^2}$

(*b*) $\kappa = \dfrac{1}{(1+t^2)^2}$ (*d*) $\mathbf{B} = \dfrac{(t^2-1)\mathbf{i} - 2t\,\mathbf{j} + (t^2+1)\mathbf{k}}{\sqrt{2}(1+t^2)}$

66.

A particle moves along the curve $\mathbf{r} = (t^3 - 4t)\mathbf{i} + (t^2 + 4t)\mathbf{j} + (8t^2 - 3t^3)\mathbf{k}$, where t is the time. Find the magnitudes of the tangential and normal components of its acceleration when $t = 2$.

Ans. Tangential, 16 ; normal, $2\sqrt{73}$