Tutorial 4

Pass up personally
on
Friday 10 pm class
7 Nov 2011

Tutorial for Chapter 4

44. If $F = x^2z + e^{y/x}$ and $G = 2z^2y - xy^2$, find (*a*) $\nabla(F+G)$ and (*b*) $\nabla(FG)$ at the point $(1,0,-2)$.

Ans. (*a*) $-4\mathbf{i} + 9\mathbf{j} + \mathbf{k}$, (*b*) $-8\mathbf{j}$

46. Prove $\nabla f(r) = \dfrac{f'(r)\,\mathbf{r}}{r}$.

51. If $\nabla\phi = 2xyz^3\,\mathbf{i} + x^2z^3\,\mathbf{j} + 3x^2yz^2\,\mathbf{k}$, find $\phi(x,y,z)$ if $\phi(1,-2,2) = 4$. *Ans.* $\phi = x^2yz^3 + 20$

54. If F is a differentiable function of x, y, z, t where x, y, z are differentiable functions of t, prove that

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \nabla F \cdot \frac{d\mathbf{r}}{dt}$$

55. If $\mathbf{A}$ is a constant vector, prove $\nabla(\mathbf{r} \cdot \mathbf{A}) = \mathbf{A}$.

58. Find a unit vector which is perpendicular to the surface of the surface of the paraboloid of revolution $z = x^2 + y^2$ at the point (1,2,5).

Ans. $\dfrac{2\mathbf{i} + 4\mathbf{j} - \mathbf{k}}{\pm\sqrt{21}}$

64. In what direction from the point (1,3,2) is the directional derivative of $\phi = 2xz - y^2$ a maximum? What is the magnitude of this maximum?

Ans. In the direction of the vector $4\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$, $2\sqrt{14}$

76. If $\boldsymbol{\omega}$ is a constant vector and $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$, prove that $\text{div}\, \mathbf{v} = 0$.

82. If $\mathbf{A} = \mathbf{r}/r$, find grad div $\mathbf{A}$. *Ans.* $-2r^{-3}\,\mathbf{r}$

91. Evaluate $\nabla \times (\mathbf{r}/r^2)$. *Ans.* $\mathbf{0}$

103. Show that $\mathbf{E} = \mathbf{r}/r^2$ is irrotational. Find ϕ such that $\mathbf{E} = -\nabla\phi$ and such that $\phi(a) = 0$ where $a > 0$.

Ans. $\phi = \ln(a/r)$

104. If $\mathbf{A}$ and $\mathbf{B}$ are irrotational, prove that $\mathbf{A} \times \mathbf{B}$ is solenoidal.

105. If $f(r)$ is differentiable, prove that $f(r)\mathbf{r}$ is irrotational.